Islamic Astronomy in China during the Yuan and Ming Dynasties

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This article is a translation of Part 2, Chapter 3 (元明時代のイスラム天文学) of Yabuuti Kiyosi 藪内清, *Chinese Astronomy and Calendrical Sciences* (中国の天文暦法, in Japanese), Tokyo (Heibonsha) 1969, 2nd. ed. 1990. The translation has been supplemented by references to recent publications in western languages and explanations of names and concepts which may be unfamiliar to non-sinologists. Incidentally the order of the sentences in the original Japanese and the place of footnotes have been changed; on two occasions, indicated by footnotes, complete paragraphs have been rewritten.
An appendix by the translator presents as alphabetically numbered notes to the main text the Chinese characters for all titles of books and names of technical concepts which have been translated into English. Furthermore, it gives brief descriptions of the most important persons and primary sources occurring in the article and of the Chinese method of counting the years by reign periods and/or the sexagesimal cycle.
Finally, an epilogue presents a sketch of current developments in research on the *Huihui li* and its relation with Arabic and Persian astronomical works. Note that all through the article names of Chinese, Japanese and Korean persons occur in the usual order, i.e. with the surname preceding the given name.

After the Mongols had built up a huge empire extending over Europe and Asia, they founded the Yuan Dynasty in mainland China and established for the first time in Chinese history the rule over complete China by a foreign people. The Mongol ruler Khubilai Khan named the country Yuan in the year 8 of his reign period Zhi Yuan (1271 A. D.), but already since the time of Chinggis Khan the Mongols had adopted Chinese culture and, as far as calendars are concerned, had employed the *Daming li* of the Jin Dynasty. According to the *Calendar Annals* of the *Yuanshi*, the official history of the Yaun dynasty, the *Daming li* was in use in the fifteenth year of Chinggis' western expedition, a year *geng-chen* (seventeenth year of the sexagesimal cycle, 1220).[1] At that time Chinggis

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[1] According to the *Summary of Historical Excellent Procedures* (歴代長術輯要) by the Qing scholar Wang Yuezhen 汪曰楨, the revision of the *Daming li* by Zhao Zhiwei 趙知微 of the Jin Dynasty had been used from the tenth year of the reign of Chinggis Khan (1215) onwards. In that year the Mongol army marched southwards and occupied Zhongdu 中都 (Beijing), the capital of the Jin Dynasty.

stayed in Samarkand and his renowned high official Yelü Chucai 耶律楚材 compiled a calendar which took into account the difference in geographical longitude between that locality and mainland China. This was the *Western Expedition Calendar with Epoch Year Geng-Wu* (i.e. 1210), which is recorded in the *Yuanshi*.[2] Except for the small correction due to the difference in geographical longitude, it is identical to the *Daming li*. In the section on the lunar position we read:[3]

> "Making Xun-si-gan cheng 尋斯干城 the standard, we put the distance in *li*s on the counting board and multiply it by 4359. Moving the (decimal) position backwards, we divide by 10000 and make *fens*. These are called the *li difference*."

From this it can be seen that Xun-si-gan cheng (Samarkand) was taken as the central locality of reference for all calculations of date and time. The *li difference* mentioned above is the correction for the distance from mainland China. In Chapter 9 of the *Writings while the Plough is Resting*[a] by Tao Zong-yi 陶宗儀 of the Yuan Dynasty, it is stated that Yelü Chucai, who was well aware that the calculation by means of "western" astronomical tables of planetary positions in particular was very accurate, also wrote the *Madaba li*[b] based on "western" methods. Although this description is very brief and the actual contents of the work are unknown, there is no doubt that it was written on the basis of Islamic astronomical tables.

After various Islamic countries had come under Mongol power and the Mongol rule over mainland China had been established, Muslim astronomers were invited to the Yuan capital Beijing. Among them the most famous scholar was Zhamaluding 札馬魯丁,[4] who compiled the *Wannian li* ("Ten Thousand Year calendar") in the year 4 of Zhi Yuan (1267). Although the contents of this calendar are unknown, it is stated in the *Yuanshi* that Khubilai Khan "distributed it in considerable amounts". Under the reign of Hülegü Khan, who had captured Baghdad in 1258 and had overthrown the Abbasid dynasty, a large astronomical observatory was constructed in Maragha, close to the north-western border of Iran. The supervisor of this observatory, Naṣīr al-Dīn al-Ṭūsī, compiled an im-

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[2] In the *Calendar Annals* of the *Yuanshi* the western expedition of Chinggis Khan (Tai Zu 太祖) is mistakenly called "the western expedition of Ögödei" (Tai Zong 太宗, son and successor of Chinggis, 1229–1241). Yelü Chucai's *Westward Expedition Calendrical Tables with Epoch Year Geng-Wu* (進西征庚午元曆表) in Chapter 8 of his *Collected Works of Zhanran Jushi* (湛然居士集) confirms that this calendar was made when Chinggis Khan stayed in Samarkand and states that it was "prepared for use in the temporary palace", i.e. presented for temporary usage.

[3] Note that a *li* 里 is a measure of length somewhat less than 600 meter, and a *fen* 分 is a ten-thousandth. (BvD)

[4] According to H. H. Howorth, *History of the Mongols*, vol. 3, London 1888, pp. 137–138, the Great Khan Möngke (Xian Zong 憲宗, grandson of Chinggis, 1251–1259) studied Euclidean geometry and planned to construct an observatory in the Mongol capital Karakorum. He consulted a Jamāl al-Dīn from Bukhara, but the observatory was never realised. L. P. E. A. Sédillot, *Prolégomènes des tables astronomiques d'Oloug Beg*, Paris 1847, pp. C–CII, relates that Khubilai Khan introduced western astronomers to China after he had subdued the entire country. Then, in 1280, Guo Shoujing received the astronomical tables by Ibn Yūnus from a certain Jamāl al-Dīn and studied them in detail. The Jamāl al-Dīn in both accounts can be assumed to be identical with Zhamaluding.

portant set of astronomical tables called the *Īlkhānī Zīj*.[5] A theory that Zhamaluding was once connected to the observatory in Maragha is doubtful.[6] But there did exist an interchange between China and Maragha and, more generally, Persian astronomers and scientific books written in Persian were brought to China. As we will see below, these astronomers also built astronomical instruments of Islamic type.

Although the traditional Chinese astronomers were to some extent stimulated by the introduction of western (i.e. Islamic) astronomy, they continued to make improvements in their calendars along traditional lines. The project of correcting the errors in the *Daming li*, which, as we have seen above, had been in use since the time of Chinggis Khan, made rapid progress after the capital Lin'an 臨安 (present-day Hangzhou) of the Southern Song Dynasty had been occupied in the year 13 of Zhi Yuan (1276) and the knowledge of its astronomers and books about astronomical calendars were obtained. Xu Heng 許衡, Wang Xun 王恂 and Guo Shoujing 郭守敬, who had served the Yuan Dynasty before that time, played a central role in mobilizing the astronomers from north and south and finally in the year 17 of Zhi Yuan (1280) a new calendar was completed. This was the *Shoushi li*, known as one of the best Chinese calendars. Almanacs based on it were widely distributed starting from the following year.

Because the *Shoushi li* was entirely a product of the Chinese tradition, Islamic astronomical knowledge cannot be recognized in it; it was strongly influenced especially by the *Tongtian li* of the Southern Song Dynasty. Islamic influence can be seen only in the equipment of the astronomical observatory and in the observational instruments built by Guo Shoujing, with which accurate observations of summer and winter solstices were made for the production of calendar works. In the year 8 of Zhi Yuan (1271) an Islamic Astronomical Observatory was established separately from the observatory of the Chinese scholars. Since there was not much interchange between the two institutions, the influence of Islamic astronomy never became very significant.

After the Yuan empire had disintegrated, the Ming Dynasty, which was once again genuine Chinese, was established in 1368. Under this dynasty the *Shoushi li* continued to be distributed with only very insignificant revisions under the new name *Datong li*. Also the organizational structure of the Islamic Astronomical Observatory was maintained and the research of the Muslim astronomers continued as before. However, on the whole the lively activities that had been seen during the Yuan Dynasty came to an end. In general it can be said that as far as astronomy is concerned the Ming succeeded to the tradition of the Yuan with little enthusiasm. Only after the arrival of the Jesuits towards the end of the

[5] The *Huihui li* (回回曆, "Islamic calendar") to be discussed below was translated during the Ming Dynasty. Since Islamic astronomy did not change that much during the Yuan and Ming dynasties, the present author was much interested in the relation between the *Huihui li* and the *Īlkhānī Zīj*. From correspondence with Prof. E. S. Kennedy of the American University of Beirut the present author learned that the numerical values in the *Huihui li* do not agree with those in a manuscript of the *Īlkhānī Zīj* in the Bodleian Library in Oxford.

[6] This theory is refuted in Yamada Keiji 山田慶児, *Way to the Shoushi li* (授時暦の道), Tokyo (Misuzushobō) 1980, which states that Zhamaluding served Möngke in Karakorum before the foundation of the observatory in Maragha. According to Howorth (see footnote 4) the observatory in Maragha was built by Naṣīr al-Dīn al-Ṭūsī on the order of Hülegü Khan, founder of the Ilkhan empire; he does not mention Jamāl al-Dīn in this context.

Ming Dynasty a translation of the *Treatise on Calendrical Science of the Chong Zhen reign period*[c] was made and the situation changed completely. However, it was only after the beginning of the Qing Dynasty that this translation started to bear fruit. Since we have discussed the introduction of European astronomy into China elsewhere, we will here concentrate our attention on Islamic astronomy.[7]

1. Astronomical Instruments of Islamic Origin

As has been mentioned above, the Islamic Astronomical Observatory was established under the reign of Khubilai Khan in the year 8 of Zhi Yuan (1271), and Zhamaluding was appointed its first director, called *ti-dian* 提點. Four years earlier this excellent scholar of Persian origin had written the *Wannian li* (see above) and had presented seven instruments of western type to the emperor. In Chapter 48 of the *Astronomical Annals* of the *Yuanshi* transcriptions and Chinese translations of the original Persian names of these instruments are given and their structure is described. Furthermore, in the section *Officials belonging to the Astronomical Bureau*[d] in Chapter 7 of the *Annals of the Yuan Imperial Library*[e] the names of 23 written works and three instruments brought from the west can be found. Since also in this case the original names are in Persian, it may be assumed that Islamic astronomy or Islamic science in general was introduced to China through Persian scholars. Of course, politics and military affairs in Islamic society were in the hands of Arabs, but in the domain of science Persians were more active. Since Islamic science was transmitted to Yuan China through the Ilkhan empire, especially in this area the involvement of Persians was very deep.

In the section *Western Instruments*[f] of Chapter 48 of the *Astronomical Annals* of the *Yuanshi*, the names and usage of the seven instruments presented by Zhamaluding are summed up as follows:

1. *zá-tū-hā-là-jí*[g], in Chinese "armillary sphere[h]"
2. *zá-tū-shuò-bā-tái*[i], in Chinese "instrument for measuring the stars of the celestial vault[j]"
3. *lǔ-hā-má-yì-miǎo-wā-zhǐ*[k], in Chinese "gnomon building for the vernal and autumnal equinoxes[l]"
4. *lǔ-hā-má-yì-mù-sī-tǎ-yú*[m], in Chinese "gnomon building for the winter and summer solstices[n]"
5. *kǔ-lái-yì-sǎ-má*[o], in Chinese "celestial globe[p]"
6. *kǔ-lái-yì-à-ér-zǐ*[q], in Chinese "geographical map[r]"
7. *wù-sù-dū-ér-là*[s], undetermined, in Chinese "instrument for the hours of day and night[t]"

Besides, the *Astronomical Annals* of the *Yuanshi* describe the structure of each of the instruments.

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[7] The introduction of European astronomy into China is discussed in Yabuuti Kiyosi, *Chinese Astronomy and Calendrical Sciences* (中国の天文暦法, in Japanese), Part 1, Chapter 7. For the *Shoushi li* and the *Datong li* the reader is referred to Part 1, Chapter 6 of the same work.

In investigations by Tasaka[8] and Hartner[9] the identification of the instruments has been attempted. According to Hartner, the names of the instruments listed above are transliterations of the following Arabo-Persian words:

1. *dhātu 'l-ḥalaq(i)* ("the owner of the rings"), i.e. an armillary sphere.
2. *dhātu 'l-shuʿbatai(ni)* ("instrument with two legs"), i.e. a parallactic ruler, an observational instrument found in Ptolemy's astronomical works and later called *triquetrum*. It is used to measure the zenith distance of a heavenly body at the time of its culmination.
3. *rukhāma-i-muʿawwaj*
4. *rukhāma-i-mustawī*
5. *kura-i-samāʾ*, a celestial globe, in China called *hun-xiang* 渾象
6. *kura-i-arḍ*, a terrestrial globe
7. *usṭurlāb*, an astrolabe.

There were instruments in China resembling the armillary sphere and the celestial globe, but all others were completely new. Since in China the theory that the earth is spherical had not been established, no terrestrial globes were produced. The *triquetrum* and the astrolabe were instruments of Greek origin and were widely used in the Islamic world, the astrolabe in particular being employed in navigation and other areas.

Hartner interprets the names of instruments 3 and 4 as follows:

3. sundial for unequal hours
4. sundial for equal hours.

As we have seen above, for instrument 3 the *Astronomical Annals* of the *Yuanshi* gives "in Chinese: gnomon building for the vernal and autumnal equinoxes", for instrument 4, "in Chinese: gnomon building for the winter and summer solstices". Hartner assumes that "for the vernal and autumnal equinoxes" refers to *equal hours* and "for the winter and summer solstices" to *unequal hours*. Consequently he argues that the Chinese translations of the names of instruments 3 and 4 were exchanged. The equal hours divide a day into 24 equal parts as in the modern system of time measurement. The unequal or *seasonal* hours, which were often used in Islamic astronomy, divide the period between sunrise and sunset in 12 equal parts and hence vary all through the year. Although instruments 3 and 4 leave some more problems to be solved besides the confusion of the Chinese names, the identification of the seven instruments as presented above is generally

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[8] Tasaka Kōdō 田坂興道, "About an aspect of Islamic culture moving eastwards" (東漸せるイスラム文化の一側面について, in Japanese), *Shigaku zasshi* 史學雜誌 53 (1942) nos. 4 (pp. 401–466) and 5 (pp. 555–605). This paper deals not only with astronomical instruments, but also with the identification of the scientific books mentioned in the *Annals of the Directorate of the Yuan Imperial Library*. Tasaka's achievements were collected in *The introduction and development of Islam in China* (中國における回教の傳來とその弘通, in Japanese), vol. 2, Tokyo (Tōyō Bunko) 1964. Part of his work was published in English: *An Aspect of Islam Culture Introduced into China*, Tokyo (Tōyō Bunko) 1957.

[9] Willy Hartner, "The astronomical instruments of Cha-ma-lu-ting, their identification, and their relations to the instruments of the observatory of Marāgha", *Isis* 41 (1950), pp. 184–194, reprinted with additional notes in Willy Hartner, *Oriens-Occidens*, Hildesheim 1968, pp. 215–226. The English translations of the names of the instruments as given above were partially taken from Hartner's article.

correct.[10]

Tasaka has also investigated the books and instruments mentioned in the *Annals of the Directorate of the Yuan Imperial Library* (see above). Among the books there were works representing Greek science such as Ptolemy's *Almagest* and Euclid's *Elements*, written in Persian. It can be assumed that both the books and the instruments were used by the western astronomers at the Islamic Astronomical Observatory.

The Islamic instruments in particular exerted a considerable influence on Chinese astronomers. For the compilation of the *Shoushi li* new instruments were built and observations with these instruments were carried out. Guo Shoujing, who was also involved in irrigation projects as the "Vice-Director of Waterways[u]", became the centre of these activities. The "Star Observation Platform[v]" from the Yuan period, which still exists in the city of Gaocheng 告成, south-east of present-day Luoyang 洛陽, was constructed in this period.[11] In spite of its name, it was a device for measuring the shadow of the sun at the time of its culmination. Thus, as far as its function is concerned, it was a *gnomon*, in China since ancient times called *bi* 髀 or *biao* 表. Normally, it suffices for a gnomon to have a pole, set up vertically on the ground, and a scale, a so-called "Gnomon Shadow Template" (土圭 *tu-gui*), which measures the shadow of the pole. However, the observational device in Gaocheng is a huge building and if we compare it with the Islamic structures which are called *masonry instrument*, it may be regarded as a result of Islamic influence.[12]

According to the *Summary of the Affairs of Famous Officials of the Yuan Dynasty*[w], Guo Shoujing made 13 astronomical instruments: a Simplified Instrument[x], a Lofty Gnomon[y], a Pole-Observing Instrument[z], a celestial globe[aa], an Ingenious Armillary Sphere[ab], an Upward-Looking Instrument[ac], a Vertical Revolving Circle[ad], a Verification Instrument[ae], a Shadow Definer[af], an Observing Table[ag], an Instrument for Observing Solar and Lunar Eclipses[ah], a Star-Dial[ai], and a Time-Determining Instrument[aj].

Furthermore, he is said to have produced a True Direction Table[ak] and eight other observational instruments or diagrams. The *Astronomical Annals* of the *Yuanshi* contain

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[10] A more exhaustive analysis of the available material, taking the descriptions of the instruments as the starting point rather than their names and arriving at various new conclusions, has been made in: Miyajima Kazuhiko 宮島一彦, "A new identification of the Islamic astronomical instruments described in the Yuan dynastic history" (元史天文志記載のイスラム天文儀器について, in Japanese), in *Science and Skills in Asia* (東洋の科学と技術, Festschrift for Prof. Kiyosi Yabuuti), Kyoto (Dōhōsha) 1982, pp. 407–427.

[11] Gaocheng is the old Yangcheng 陽城, which used to be called "the centre of the earth". The Star Observation Platform is part of the Tower of the Duke of Zhou for Measuring the Sun's Shadow (周公測景臺). A detailed investigation of the remainders of observatories from the Yuan Dynasty can be found in: Dong Zuobin 董作賓, Liu Dunzhen 劉敦楨 and Gao Pingzi 高平子, *Report of an Investigation of the Tower of the Duke of Zhou for Measuring the Sun's Shadow* (周公測景臺調査報告, in Chinese), Changsha (Commercial Press) 1939. The Tower of the Duke of Zhou is also discussed in Joseph Needham e.a., *Science and Civilisation in China*, vol. 3, Cambridge 1959, pp. 294 ff.

[12] See Yabuuti Kiyosi, "Islamic astronomical observatories and observational instruments" (イスラムの天文台と観測器械, in Japanese), in *Crossroads of civilization* (文明の十字路), Tokyo 1962, pp. 144–155. This article has been included in *Chinese Astronomy and Calendrical Sciences* (中国の天文暦法), as Part 2, Chapter 5. The standard work in English on Islamic observatories is Aydın Sayılı, *The observatory in Islam*, Ankara 1960, 2nd ed. 1988.

rather detailed explanations concerning the Simplified Instrument, the Upward-Looking Instrument, the Direction-Determining Table, the Shadow Definer, the Observing Table, and, not mentioned in the list above, the Lantern Clepsydra in the Grand Illumination Hall[al] and the Measuring Scale and Gnomon[am]. It seems that various of these instruments show traces of Islamic influence. For instance, it is said that the Simplified Instrument was devised by Naṣīr al-Dīn al-Ṭūsī at the observatory in Maragha, but in fact it seems to have been built even earlier by the Spanish-Muslim scholar Jābir ibn Aflaḥ (born c. 1130). In the early eighteenth-century observatory of the Maharaja Jai Singh in Jaipur (India), which was strongly influenced by Islamic observatories, it exists under the name *krāntivṛtti-valaya-yantra.* The Simplified Instrument of Guo Shoujing was a somewhat simpler form of the latter.[13] Likewise, the Upward-Looking Instrument, a sundial in the form of an upward-looking bowl, generally called a *scaphe sundial*[an], was not a traditional Chinese instrument.[14]

We will omit explanations concerning the other instruments, but may conclude that the influence of Islamic astronomy on Chinese instruments was considerable. On the other hand, as we have discussed above, no Islamic elements are present in the contents of the *Shoushi li.* The Simplified Instrument still exists in the Purple Mountain Observatory in Nanjing. The *scaphe sundial*, being a relatively simple instrument, was made in China as well as in Korea and a copy also exists in the National Science Museum in Tokyo.

The Islamic Astronomical Observatory, which had been established with Zhamaluding as its first director, continued to exist on a somewhat smaller scale during the Ming Dynasty. For detailed information concerning the activities of the Muslim scholars, centred around the Astronomical Observatory, we would like to refer to an article by Tasaka.[15]

2. Islamic Astronomical Tables

Concerning the *Wannian li*, composed by Zhamaluding during the early Yuan Dynasty, the preface by the Ming scholar Song Lian 宋濂 contained in the *New Treatise on the Heavenly Bodies*[ao], states the following:

> "As we hear, the Western country is far out of the reach of the Great Wall. Already during the Yuan Dynasty this country was occupied. There was a person Zhamaluding who presented the *Wannian li*. His method of measuring the heaven

[13] In Needham's *Science and Civilisation in China*, vol. 3 (see footnote 11), pp. 369–372, the instruments appearing in the *Astronomical Annals* of the *Yuanshi* are identified and a particularly detailed description of the Simplified Instrument is presented. See also: Joseph Needham e.a., *The Hall of Heavenly Records. Korean astronomical instruments and clocks* 1380–1780, Cambridge 1986. The English translations of the names of the instruments as presented above were mostly taken from Needham's works. (BvD)

[14] A picture of a *scaph sundial* can be found in Needham's *Science and Civilisation in China*, vol. 3 (see footnote 11), opposite p. 301 (Fig. 123), or in Jeon Sang-woon 全相運, *Science and Technology in Korea: Traditional Instruments and Techniques*, Cambridge Mass. 1974, p. 46 (Fig. 1.11).

[15] Tasaka Kōdō, "The introduction of the western calendar in the east and the fate of the Islamic Calendar" (西洋暦法の東漸と回々暦法の運命), Tōyō Gakuhō 東洋學報 31 (1947), pp. 141–180.

> uses only 12 zodiacal signs and divides it into 360 degrees. As for the theory of the 28 lunar mansions, it is as if he had not heard of it. The agreement of his predictions of very thin solar and lunar eclipses with those of the Chinese, is because the principles are the same."

Usually calendrical tables assigning dates some tens or hundreds of years in advance, were called a "Ten Thousand Year" (*wan-nian* 萬年) calendar. However, the *Wannian li* by Zhamaluding seems to have been a computational work containing calculations of solar and lunar eclipses and would nowadays be called astronomical tables. Islamic astronomy, which was based on Greek astronomy, did not use the 28 lunar mansions for indicating the positions of the heavenly bodies, but took the 12 signs of the zodiac as its reference system and divided the heavenly cycle into 360 degrees. The preface by Song Lian states clearly that the *Wannian li* of Zhamaluding was written on the basis of astronomy of this type.

It seems that Chinese translations of two or three Islamic astronomical works were made during the Yuan Dynasty, but these have been lost entirely. According to the account in the *Huihui lifa* (回回曆法, "Islamic Calendar Method") in the *Mingshi*, a Chinese translation of the *Huihui li*, the "Islamic Calendar", had already been made in the beginning of the Yuan. However, because the western astronomers in the Yuan and Ming Dynasties performed their calculations exclusively on the basis of the original work, the Chinese translation was hardly ever used. Therefore errors were produced during its transmission and it became unsuitable for perusal. Then the compilers of the *Mingshi* say:

> "We have extensively visited the descendents of the specialists and have investigated the original book. We filled in its omissions and corrected its errors. Now we make the *Huihui lifa*, which we write down in a book."

Thus it can be assumed that when the *Huihui lifa* was compiled during the Qing Dynasty, the relevant data existing at that time were used.

Essentially identical to the *Huihui lifa* in the *Mingshi*, but more detailed, are the seven chapters of the *Qizheng tuibu* (七政推步, "On the Motion of the Seven Planets"), compiled by Bei Lin 貝琳 during the Ming Dynasty. Furthermore, the *Huihui li* has been handed down in the recension of the *The Records of King Sejong* (世祖實録 *Sejong Sillok*) of the Korean Yi Dynasty. Chapters 156 to 163 of this work contain calendars brought from China; of the total of eight chapters, the three chapters of the Inner Book deal with the *Shoushi li*, the remaining five chapters of the Outer Book with the *Huihui li*. According to the introductory part of Chapter 156, the *Huihui li* as it had been brought from China was revised on imperial order by the famous scholars Yi Sun-ji 李純之 and Kim Tam 金淡 of the Sejong empire.

We will first discuss the *Qizheng tuibu*. In the colophon of this work by Bei Lin we read:

> "This kind of book we never had in ancient times, but in the year 18 of the reign of Hong Wu (the first Ming emperor, 1385) a foreigner was naturalized and pre-

> sented a calendar based on dustboard methods. It gave predictions of occultations of the moon and the five planets and its title was "Longitudes and latitudes". Then Yuan Tong 元統 of the Calendar Bureau took out the dustboard methods and translated them into "Chinese calculation methods" (漢算 *han-suan*). Thus the book circulated for the first time in China."

Furthermore, it is stated that Bei Lin revised the remaining fragments of the book and completed it in the year 13 of Cheng Hua (1477). The term "calendar based on dustboard methods" indicates that the work used Arabic numerals, which Yuan Tong of the Calendar Bureau changed into calculation methods of Chinese type, i.e. *han-suan*. The *Summaries of the Complete Books of the Four Imperial Divisions*[ap], a detailed description of the contents of the imperial archives from the Qing Dynasty, points out that the formation of the *Huihui lifa* in the *Mingshi* and that of the *Qizheng tuibu* are different. It concludes from the statement "The completed western years until the year *jia-zi* of Hong Wu accumulate to 786" found in the *Qizheng tuibu* that this book had already been translated in the year 17 of Hong Wu (the year *jia-zi*, first year of the sexagesimal cycle). This would contradict Bei Lin's account that a foreigner presented the work in the year 18 and that it was then translated into Chinese by Yuan Tong. However, the year *jia-zi* of Hong Wu was only adopted as a convenient year for the calendar calculations, and was by no means the year of completion of the book itself. The above statement rather indicates that the book originated *after* the year *jia-zi*. In the *Summaries of the Complete Books of the Four Imperial Divisions*, the *Qizheng tuibu* is praised as follows: "The *Mingshi* discusses the outline of the methods in the *Huihui li* to some extent, but the *Qizheng tuibu* is the original book and describes the details in clarity", indicating that it fills in the missing parts of the *Huihui lifa* in the *Mingshi*.

The *Huihui li* in the *True Records of King Sejong* is in various respects somewhat different from the *Qizheng tuibu*. For instance, in corresponding parts with identical contents the wording does not precisely agree. Furthermore, in the *Qizheng tuibu* the complete computational rules are contained in the first chapter, whereas the tables (立成 *li-cheng*) necessary for the computations are put together starting from the second chapter. In the *True Records*, on the other hand, the tables are intermingled with the explanatory text describing their use. Thus, in comparison with the *Qizheng tuibu*, its arrangement is far more convenient. In fact, we can say that the *True Records* have handed down a revision of the Chinese translation of the *Huihui li* in the style of a traditional Chinese calendar.

The versions of the *Huihui li* in the *Mingshi*, the *Qizheng tuibu*, and the *True Records* are basically identical as far as their astronomical contents are concerned. There are some differences in the wording, arrangement, etc. and, for only few parts of the text, in the level of detailedness. In particular, different from the other two works, in the *Huihui lifa* in the *Mingshi* the section concerning the star table is highly incomplete and very brief. On the other hand, the *Mingshi* gives information related to the method of constructing the tables which cannot be found in the other two versions. The *True Records* are particularly superior as far as the arrangement of the material is concerned. Since in the *Qizheng tuibu*, which contains approximately the same amount of material as the *True Records*, the introductory part has remarkable characteristics, we will now briefly discuss the sections of this part one by one.[16]

"*Explaining the parameters* (用數 *yong-shu*)" This section states that the "heavenly circle" (周天 *zhou-tian*) is divided into 360 degrees. This is equal to the modern method, but fundamentally different from the traditional Chinese one, in which the number of degrees of the heavenly circle followed the length of the year.

"*Explaining the accumulated years* (積年 *ji-nian*) *of the Huihui li*" The "Western Arabic Year[aq]" is taken to be the year *ji-wei* (56th year of the sexagesimal cycle) of the reign period Kai Huang of the Sui Dynasty (599 A. D.), and with this as the epoch 786 years are counted till the year *jia-zi* of Hong Wu. As has been pointed out by Dr. Kuwahara, there is a mistake in this calculation.[17] Counting according to the purely lunar Islamic calendar, the year *jia-zi* of Hong Wu (1384) corresponds precisely to the year 786 Hijra. However, in the *Huihui li* the Hijra epoch year is assumed to be the year *ji-wei* of Kai Huang (599), which was mistakenly obtained by tracing back 786 *solar* years. Correct is 622, the year *ren-wu* of the reign period Wu De of the Tang Dynasty. In the article on Bei Lin in *Biographies of Mathematicians and Astronomers*[ar], which appeared in 1799, Ruan Yuan 阮元 affirms the opinion of the early-Qing astronomer Wang Xichan 王錫闡 that "The epoch of the dustboard methods falls in a year of Wu De, not in the year *ji-wei* of Kai Huang". However, he makes a mistake by asserting that two epochs were used, the year *ji-wei* of Kai Huang for the solar calendar and *ren-wu* of Wu De for the lunar calendar. The first day of Muharram, first month in the Islamic calendar, of the Hijra epoch year corresponds either to Thursday 15 July or to Friday 16 July 622 in the Julian calendar, the first date following the method of calculation of the astronomers, which makes the sun-moon conjunction the first day of each month, the second date following the civil calendar, which determines the first day by the first sighting of the lunar crescent. Both the *Mingshi* and the *True Records* state clearly that the beginning of the day in the Islamic calendar is noon.

"*Explaining the number of days of the zodiacal signs* (宮 *gong*)" In the section concerning the solar calendar, the months are called by the names of the twelve zodiacal signs. Depending on the varying velocity of the solar motion, Aries (whose beginning is reached by the sun at the time of the vernal equinox) has 31 days, the longest month Cancer 32 days, and the shortest months Sagittarius and Capricorn 29 days. The months according to this particular type of solar calendar are called "immovable". The length of a year is 365 days in an ordinary year; in a leap-year one day is added to the twelfth month,

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[16] The following text and Sections 3, 4 and 5 below mainly follow the present author's article "Explanation of the *Huihui li*" (回回曆解, in Japanese), Tōhō Gakuhō 東方學報 36 (1964), pp. 611–632. A summary in English of the same material has appeared as "The Influence of Islamic astronomy in China", in *From Deferent to Equant. A volume of studies in the history of science in the ancient and medieval Near-East in honor of E. S. Kennedy*, New York (Annals of the New York Academy of Science 500) 1987.

[17] Kuwahara Jitsuzō 桑原隲蔵, *Collection of Treatises on the History of Oriental Civilization* (東洋文明史論叢, in Japanese), Tokyo (Kōbundō shobō) 1933, p. 462. However, as we will discuss below, the mistake had already been noticed by the early-Qing scholar Wang Xichan.

Pisces, making the year 366 days.[18]

"Explaining the large and small months and their names with the original sounds" As the names of the months in the Islamic lunar calendar are given transliterations of the names of the Persian solar months. This seems to prove that the *Huihui li* was derived from an astronomical work that originated in Persia. The transliteration into Chinese and the original Persian are as follows:

1)	*fǎ-ér-wò-ér-dīng*[as]	Farwardīn	7)	*liè(bié?)-hēi-ér*[ay]	Mihr
2)	*ā-ér-de-bì-xī-shì*[at]	Ardībihisht	8)	*ā-bān*[az]	Ābān
3)	*hǔ-ér-dá*[au]	Khurdādh	9)	*ā-zá-zì*[ba]	Ādhar
4)	*tí-ér*[av]	Tīr	10)	*dá-yì*[bb]	Day
5)	*mù-ér-dá*[aw]	Murdādh	11)	*bā-hā-màn*[bc]	Bahman
6)	*shā-hé-liè-wò-ér*[ax]	Shahrīwar	12)	*yì-sī-fān-dá-ér-má-de*[bd]	Isfandārmudh

The months of the lunar calendar are called "movable". The first month is large (30 days), the second month small (29 days), and further large and small months alternate. Thus an ordinary year has 354 days; in a leap-year one day is added to the twelfth month, making this month 30 days and the year 355. The intercalation method of the lunar calendar inserts 11 leap-years in every 30 years as we will discuss below.

"Explaining the numbers of the week days (七曜 *qi-yao*) *and their names with the original sounds"* Taking Sunday as 1 and ending with 7 for Saturday, the weekdays are listed with transliterations of the Persian.

"Explaining the method of intercalation (閏法 *run-fa*)" This section is divided into paragraphs *Intercalary Days of the Zodiacal Signs*[be], *Intercalary Days of the Lunar Months*[bf], and *Chinese Intercalary Months*[bg]. First, in the paragraph *Intercalary Days of the Zodiacal Signs* the method of calculating the leap-years in the solar calendar and the week day of the first day of Aries (the vernal equinox) are explained. For the length of the year the value $365\frac{31}{128}$ days is adopted. The years are counted starting from the vernal equinox of the epoch year. However, this vernal equinox did not coincide with the beginning of the day, but occurred $\frac{15}{128}$ days after noon. In the *Mingshi* the numerator of this fraction, 15, is called the "intercalation constant" (閏應 *run-ying*), a technical term taken from Chinese calender systems. Furthermore, the addition of 5 in the calculation of the week day of the vernal equinox is explained in the *Mingshi* as "The table of zodiacal signs begins from Tuesday, day 3, therefore we add 5". Assuming that the vernal equinox at epoch indeed took place on a Tuesday (day 3), it is very hard to understand why 5 should be added.

Next, the paragraph *Intercalary Days of the Lunar Months* shows the calculation of

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18 In Persia a true solar calendar based on the vernal equinox had been in use since the reign of the Saljuq sultan Jalāl al-Dīn Malik-shāh (1073–1092). In this calendar the month names were the standard Persian ones, listed in the *Qizheng tuibu* under the lunar calendar (see below). Usually the months were given 30 days each, leaving 5 or 6 extra days at the end of the year, but some astronomers are said to have distributed the days over the months as in the *Qizheng tuibu*, i.e. based on the entry of the sun in the zodiacal signs. For more information the reader is referred to S. H. Taqizadeh, "Various eras and calendars used in the countries of Islam", *Bulletin of the School of Oriental and African Studies* 9 (1937–39), pp. 903–922 and 10 (1939–1942), pp. 107–132, and the article "Taʾrīkh" in the *Encyclopaedia of Islam*, new edition, Leiden (Brill) 1960–. (BvD)

the leap-years in the lunar calendar. The synodic month adopted here is $29\frac{191}{360}$ days, so that a year of 12 months becomes $354\frac{11}{30}$ days. Therefore we may insert 11 intercalary days in every 30 years. By the way, in the above calculations according to the solar and lunar calendars, the number of years from the epoch to the time of calculation is in both cases called the "Western Completed Years[bh]" (lit. "western accumulated years before the (current) year"). Since these should be counted separately for the two calendars, there is clearly some confusion.

Finally, the paragraph *Chinese Intercalary Months* shows the calculation of the leap-months in the luni-solar calendar, which inserts 123 intercalary months during 334 years. Calculated with this ratio, the length of the solar year is only slightly different from the above-mentioned value. In the double-column notes after the paragraph title we find the number of accumulated years from the year *jia-zi* of Zhi Yuan of the Yuan Dynasty (1264) till *jia-zi* of Hong Wu. Although on first sight it appears that the year *jia-zi* of Zhi Yuan is taken as the epoch, in the calculations the epoch is assumed to be 137 years earlier than that, namely the year 5 of the reign period Tian Hui of the Jin Dynasty (1127). Since this is the year in which Yang Ji 楊級 compiled the *Daming li*, the paragraph *Chinese Intercalary Months* may be somehow related to this calendar. However, the lengths of years and months in the two sources do not agree. The paragraph on the Chinese intercalary months is not present in the *Mingshi* or the *True Records*.

The above has explained the particularly detailed introductory part of the *Qizheng tuibu*. In what follows, referring to all three sources, we will give an outline of Islamic astronomy, in which we use the methods of calculation which were transmitted to China, in particular those of the tables, as the starting point. Except for insignificant differences, the tabular values in the three sources are basically identical. Since the methods of calculation in Islamic astronomy are based on Ptolemy's *Almagest*, we will frequently refer to this work in our examination and will explain the motions of the heavenly bodies by means of Ptolemy's geometrical models.[19]

3. Calculation of the Solar and Lunar Position

First we will discuss the calculation of the solar position (Fig. 1). In Islamic astronomy, which was mostly based on the *Almagest*, the sun S is assumed to move uniformly on a circle around centre O. The true solar position is observed from the earth E, which is slightly removed from O. The point A is the apogee, whose longitude is called "position of the highest point[bi]". At the time of observation this longitude was $2^s29°21'$, i.e. $89°21'$. The position of the apogee is not fixed, but is assumed to advance (i.e. the longitude of A increases) at a daily rate of 0.164″. The "mean solar position[bj]" (mean ecliptical longi-

[19] Of the *Almagest* there are early French and German translations by N. Halma and K. Manitius. On the basis of these the present author has prepared a Japanese translation *The Almagest* (アルマゲスト), 2 vols., Tokyo 1949–1958; 2nd. ed. in 1 vol., Tokyo (Kōseisha) 1982. More recently, an English translation by G. J. Toomer, *Ptolemy's Almagest*, London/New York 1984, has been published. Extensive explanations of the astronomical contents of the *Almagest* can be found in O. Pedersen, *A Survey of the Almagest*, Odense 1974.

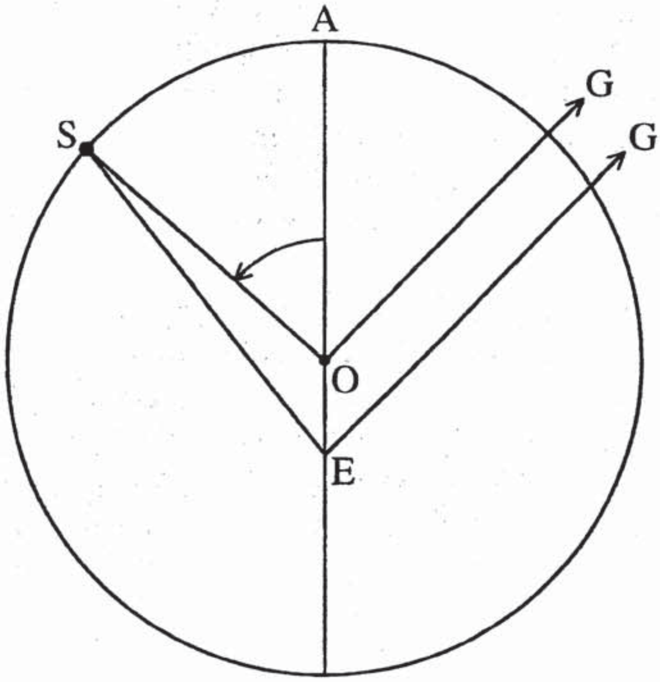

Fig. 1. Calculation of the Solar Position.

tude) increases at a daily rate of 59′8″.3. It is the angle between the reference point, the vernal equinoctial point *G*, and the sun *S*. When we subtract the position of the apogee from it, we obtain ∠*AOS*, the mean anomaly, in the *Huihui li* called the "own motion[bk]". The difference between the true and mean positions of the sun, ∠*OSE*, is called the "additive or subtractive correction for the sun[bl]" (solar equation). When we add it to or subtract it from the mean solar position, the true ecliptical longitude ∠*GES*, measured from the vernal point, is obtained.

The three tables needed for the calculation of the solar position are given in the *Huihui li*. They are the *Table of the Solar Apogee Position*[bm], the *Table of the Mean Solar Position*[bn], and the *Table of the Solar Equation and Its Differences*[bo]. The third is a single table with the angle between the apogee and the sun as the argument. However, since values for the apogee position and mean solar position need to be calculated over considerably long periods of time, the first two tables mentioned above are divided into 5 subtables: collected years (總年 *zong-nian*, "summed years"), extended years (零年 *ling-nian*, "individual years"), months, days, and zodiacal signs (宫分 *gong-fen*). Firstly, the table for days shows the motion in every number of days from 1 to 30. The tables for months and zodiacal signs display the motion in the twelve months of the Islamic lunar calendar and the Persian solar calendar respectively. Next, the table for extended years shows the motion during the 30-year cycle of the Islamic calendar, which consists of 19 ordinary years of 354 days and 11 leap-years of 355 days. According to the table, the years 2, 5, 7, 10, 13, 16, 18, 21, 24, 26 and 29 are leap-years.

Finally, the table for collected years shows the mean motions for the Hijra epoch year and, skipping the period in between, for each 30 years from 600 to 1440. Here the values for the mean solar motion are the actual mean positions at the beginnings of the respective Hijra years. The values for the apogee motion, on the other hand, must be subtracted from or added to the observed epoch value given above. Thus, the fact that the apogee motion recorded under 660 years is 0 degrees means that the solar apogee position for that year is the 89°21′ given above. In other words, the year of the observation from which the longitude of the apogee was found is 660 Hijra (1261). Since this date is very close to the year of compilation of the *Īlkhānī Zīj* by Naṣīr al-Dīn al-Ṭūsī, a close relationship between that astronomical handbook and the *Huihui li* could be conjectured. However, the numerical values in the tables of the two works turn out not to agree with each other (cf. footnote 5). It can be noted that the apogee position at the Hijra epoch is obtained by subtracting the tabular value displayed for the epoch year, $0^s10°40'$, from the actually observed value 89°21′. The result agrees quite well with the apogee position 79.4° calculated with modern methods.

By the way, if the apogee position and mean solar position are obtained for dates in the lunar calendar, the table for zodiacal signs would seem unnecessary. However, in practice values for dates in the solar calendar were calculated using the tables for the lunar calendar. Concerning this point the explanation in the *True Records* is the most detailed.

The various constants of the geometrical model in Fig. 1 can be derived from the *Table of the Solar Equation and Its Differences*. In this table the solar equation $\angle ESO$ is displayed for each degree of the argument, the distance from the apogee. The tabular differences of consecutive equation values, called "minutes of the equation[bp]", are shown in a separate column. The maximum value of the solar equation is 2°0′47″, found for a distance from the apogee equal to 92°. In general, the value of the equation is found from[20] $\tan \angle ESO = EO \cdot \sin \angle AOS / (OS + EO \cdot \cos \angle AOS)$. When we take the radius of the eccentric circle equal to 1 and calculate EO from the values given above, we find that the distance from the centre O to the earth E is 0.0351. In the *Almagest* the maximum equation is 2°23′ and consequently EO is equal to 0.0417. Compared to this, the values in the *Huihui li* are smaller. In modern astronomy EO equals twice the eccentricity of the orbit of the earth, 0.0335. Thus the values in the *Huihui li* show a significant improvement with respect to the *Almagest*.

Next we will discuss the calculation of the lunar position. As far as the irregularity of the lunar motion is concerned, already in the *Almagest* the evection was known besides the equation of centre. Thus we have to consider a model which is more complicated than that for the solar motion. The lunar "mean motion[bq]" (mean ecliptical longitude) is calculated using a daily increase of 13°10′35″, which we will call n. The mean position at epoch is given as 243°44′, although the actual value seems to have been slightly different. Following the *Almagest*, for the calculation of the irregularity a geometrical model is assumed which is remarkably different from the actual lunar motion (Fig. 2). In this model, a point B revolves on a circle with centre O. Its motion around the earth E, measured from the apogee A, is the "double elongation[br]". When we denote the daily solar motion by n' and the time (in units of a day) by t, the double elongation can be expressed as $\angle AEB = 2(n-n')t = 24°22'53''.4 \times t$. Now the moon M moves in the opposite direction on a

[20] A misprint in the formula in the Japanese original has been corrected. (BvD)

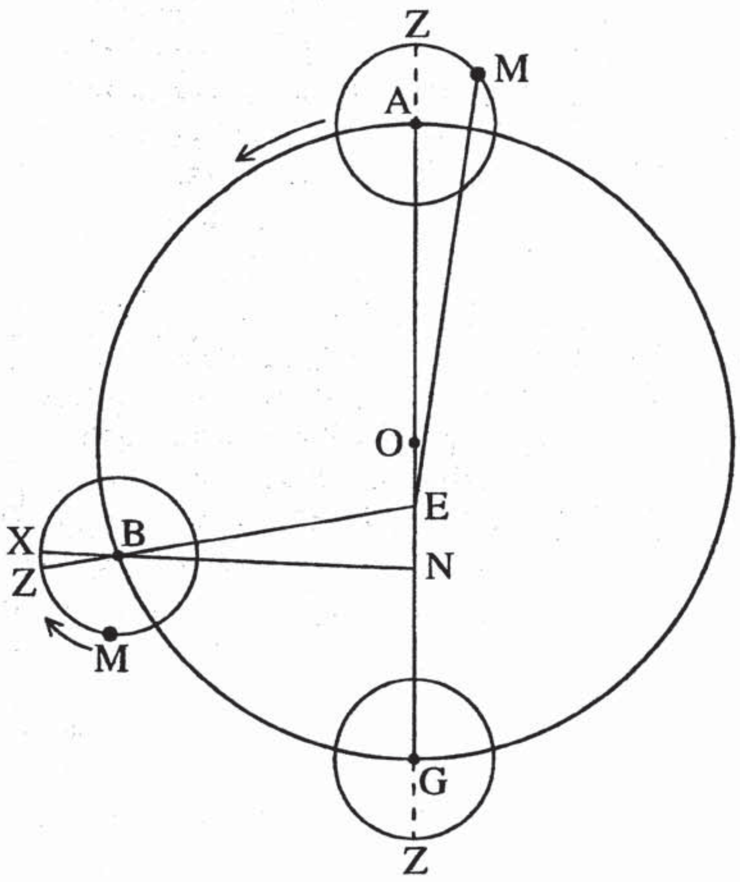

Fig. 2. Calculation of the Lunar Position.

small circle around centre *B*, the epicycle, with the anomalistic month as its period. Its motion, which amounts to 13°3′54″ per day, is called the "epicycle motion[bs]" (mean anomaly). For the above-mentioned mean motion, double elongation and epicycle motion the *Huihui li* has five types of tables just as in the case of the sun, namely for collected years, extended years, months, days and zodiacal signs.

In the *Almagest* and works succeeding it such as the *Huihui li*, the starting point of the epicycle motion is taken to be *X*, the point of intersection of the epicycle and the extension of line *NB*, where point *N*, given by *OE*=*EN*, is introduced merely in order to agree with the observations. In the *Almagest* the point *X* is called the "mean apogee of the epicycle", whereas *Z*, the point of intersection of the epicycle and the extension of line *EB*, is the "true apogee of the epicycle". In order to find the epicycle motion as measured from *Z*, the "first equation[bt]" ∠*XBZ* needs to be calculated. The *Huihui li* contains a table for the first equation with the double elongation as the argument. By adding the equation to or subtracting it from the epicycle motion, the angle between *Z* and the moon *M*, the "true epicycle position[bu]" (true anomaly), can be obtained. To determine the first equation, *OE* or *EN* needs to be known. We shall calculate these constants working backwards from the tabular values. Since in the table the value of the first equation for a double elongation of 90 degrees is 11°30′, we find *OE*=*EN*=0.161 when we take *AE* as the unit. The same table also contains a tabulation of "interpolation minutes[bv]" as a function of the double elongation, which will be explained below.

The first equation is the correction of the epicycle motion. Next, in order to find the irregularity of the lunar motion, we need the *Table of the Second Equation and Distance Degrees*[bw]. Although the distance *EB* changes with the double elongation $\angle AEB$, we ignore this fact for the time being. Considering the particular case in which point *B* in Fig. 2 coincides with the apogee *A*, the equation $\angle ZEM$ at the time when the moon reaches *M* can be obtained as a function of the true epicycle motion $\angle ZAM$ measured from *Z*. This equation is called the "second equation[bx]". When the line *EM* from the earth to the moon is tangent to the epicycle, the second equation assumes its maximum. Since in the table the largest value of the second equation, 4°50′, is found for argument 93 or 267 degrees, we can calculate the radius of the epicycle working backwards from these values. Taking *EA* as the unit, the radius is approximately 0.084. Consequently, in the case that *B* coincides with the perigee *G*, the largest value of the equation becomes approximately 7°18′. Since this is the maximum of the sum of the equation of centre and the evection, the corresponding modern value is 7°34′.

It can be noted that, for identical values of the true epicycle position, the second equation when the epicycle centre is in *G* is always larger than when it is in *A*. The difference between the two is called the "distance degrees[by]". When the epicycle centre is in between *A* and *G*, first the second equation and the distance degrees for the current true epicycle position are found from the table. Then the distance degrees are adjusted to the actual position of the epicycle centre by multiplying them by the interpolation minutes, obtained above with the double elongation as the argument, and dividing by 60. The result is added to the second equation to obtain the "definite equation[bz]". Finally, the true ecliptical longitude of the moon, called "degrees of longitude[ca]", is obtained by adding the definite equation to or subtracting it from the mean motion.

The calculation of the irregularity of the lunar motion being as above, we can note the following. The point *B* moves with the double elongation, i.e. with twice the difference between the angular velocities of the sun and the moon. Therefore, since the time at which *B* coincides with *A* is that of a new moon, *B* will coincide with *A* at the time of a full moon too. In other words, the mean lunar distances at solar and lunar eclipses are always equal to *EA* and hence have identical values. This makes the calculation of solar and lunar eclipses remarkably simple, but, at the same time, produces a result which does not agree with the actual situation.

Next the lunar latitude will be calculated, for which first of all the *Table of the Mean Position of Rāhu and Ketu*[cb] is consulted. As far as the points of intersection of the ecliptic and the inclined lunar orbit are concerned, following the terminology of Indian astronomy, it was customary to distinguish between Rāhu (羅睺 luo-hou) as the ascending node and Ketu (計都 *ji-du*) as the descending node. However, since in the above-mentioned table as well as in the explanatory text the ascending node is called *luo-ji* 羅計 or *ji-du* 計都, there seems to be some confusion about the terminology. Be that as it may, in the *Table of the Mean Position of Rāhu and Ketu* the daily retrograde motion of the nodes is assumed to be 3′11″ and tables for the motion in collected years, extended years, months, days and zodiacal signs are given. The ecliptical longitude of the ascending node at epoch is 139°15′. Using these values the position of the ascending node is calculated and by subtracting it from the previously found lunar longitude, we obtain the "distance between the moon and the node[cc]". With this distance as the argument the latitude is found from the *Table of the Lunar Ecliptical Latitude*[cd]. In this table the largest latitude value, i.e. the inclination of the inclined orbit, is given as 5°2′30″. This concludes our explanation of the solar and lunar motion.

4. Calculation of Solar and Lunar Eclipses

Since among all heavenly phenomena the astronomers of the past have made the greatest efforts towards the prediction of eclipses, this topic played a central role in astronomical computations. In the *True Records*, the first part of the Outer Book is assigned to the calculation of solar and lunar positions, whereas the whole single chapter of the second part is devoted to the method of calculating solar and lunar eclipses. The four tables needed for the prediction of eclipses are: the *Table of the Equation of Day and Night*[ce], the *Table of the Equations for Longitude, Latitude and Time*[cf], the *Table of the Solar and Lunar (Hourly) Motion and Diameter of the Shadow and the Interpolation Minutes*[cg], and the *Table for the Duration of Day and Night*[ch]. Of all four we will discuss the general outline. Because the calculation of eclipses by means of these tables is very complicated, we will not go into every detail.

Firstly, the *Table of the Equation of Day and Night* is used for the conversion from mean solar time to true solar time, i.e. it displays the equation of time. Whereas the modern equation of time is obtained by subtracting mean solar time from true solar time, the method followed in the *Almagest* and Islamic astronomical tables is quite different. Since Prof. Neugebauer has given an excellent survey of the relevant aspects of this topic, we will refer to his work.[21]

The equation of time in the *Huihui li*, which we will indicate by E, differs in the following way from the modern equation of time. Taking the equation of time e_0 at a fixed time as a reference value and indicating the value at the desired time by e, we can express the equation of time in the *Huihui li* in the form $E=e-e_0$. If we choose for e_0 the smallest value of the equation of time, E becomes always positive. Nowadays, the largest value of the equation of time during a year is approximately $+16^m23^s$ around 4 November and the smallest value -14^m20^s around 12 February, the difference being 30^m43^s. In the *Huihui li*, in which the equation of time is given as a function of the solar longitude, the smallest value of E in the table, namely 0′0″, occurs in the neighbourhood of a longitude of 321 degrees. The largest value 31′47″ is assumed when the longitude becomes 219 degrees. Considering that the equation of time changes slightly during the ages, we may say that the difference between the largest and smallest values in the *Huihui li* is close to the modern value.

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[21] O. Neugebauer, *The astronomical tables of al-Khwārizmī*, Copenhagen 1962, pp. 63–65.
See also: B. van Dalen, "On Ptolemy's table for the equation of time", *Centaurus* 37 (1994), pp. 97–153, or B. van Dalen, "al-Khwārizmī's astronomical tables revisited: analysis of the equation of time", in *From Baghdad to Barcelona. Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet*, Barcelona 1996, pp. 195–252. (BvD)

As Neugebauer points out, the Islamic astronomer al-Battānī (Syria, c. 900 A. D.) took as his reference value the equation of time e_0 on 1 March 312 B. C. (the beginning of the Seleucid Era), which was not the smallest, but rather the largest value. Therefore his E was always subtractive. On the other hand, al-Khwārizmī (Baghdad, c. 840) made the value at the beginning of the Hijra era, 15 July 622, his e_0. Since this was neither the largest value nor the smallest, E became sometimes additive and sometimes subtractive. However, by adding a minute correction to his actual values of the solar and lunar longitude, he obtained the same result as if he had adopted the maximum value for e_0, and hence his E became always subtractive. According to Neugebauer's calculations, for this purpose the solar longitude had to be increased by 49″, the lunar longitude by 11′. His analysis also shows that al-Khwārizmī's equation of time was used when converting from true solar time to mean solar time. When converting inversely from mean solar time to true solar time, the sign of E and of the longitude corrections becomes the opposite of what has been said above.

The *Table of the Equation of Day and Night* is needed in the *Huihui li* in the calculation of the true solar time corresponding to the so-called "general time of the extreme of the eclipse[ci]", the mean solar time of the conjunction. In this case E should be an additive correction, and indeed in the explanation in the paragraph *Finding the Time from Midnight till the Conjunction*[cj] it is clearly positive. Furthermore, although in the Islamic astronomical calendar the day starts at noon, in this paragraph the method of counting the hours by 1, 2, etc. starting from midnight is adopted. Therefore, if the time of the conjunction is 12 hours, the true sun is about to arrive at the meridian. The problem remains to be solved whether in the *Huihui li* the largest value of the equation of time was used as the reference value e_0 as in the case of al-Battānī, or a correction of the solar and lunar longitudes was carried out according to the method of al-Khwārizmī. However, it can be supposed that the latter method was employed, since in obtaining the solar and lunar mean motions corrections of $-1'4''$ and $-14'$ respectively are performed beforehand. Except for their sign, these values look much like those of al-Khwārizmī. In the *Huihui lifa* in the *Mingshi* the subtraction of $1'4''$ from the mean solar longitude is explained by: "As far as the subtraction of 1 minute and 4 seconds is concerned, that it might be the *li* difference that the western country is distant from China is not correct; probably it is related to the "degree constant" (度應 *du-ying*) of the last day of the solar year *ji-wei*". It is indeed clear that, besides being too small, the above correction could not derive from the difference in geographical longitude.

The calculation of solar eclipses is more complicated than that of lunar eclipses, in particular because of the necessity to consider the lunar parallax. The parallax is zero when the moon is in the zenith and becomes larger when the distance from the zenith increases. In the *Almagest* and the *Huihui li* the effect of the parallax on the ecliptical longitude and latitude is calculated, so that the relative positions of the apparent sun and the apparent moon can be obtained. For this purpose the true lunar position and the hour angle of the moon are needed, for which, however, the *Huihui li* substitutes the ecliptical longitude and the hour angle of the sun at the mean time of the conjunction. This hour angle (reckoned from midnight) is already given in the paragraph *Finding the Time from*

Midnight till the Conjunction. With the hour angle and the longitude of the sun (expressed in the twelve zodiacal signs) as a *double argument*, the parallax corrections and the resulting correction of the time of the middle of the eclipse are displayed in the *Table of the Equations for Longitude, Latitude and Time*. The use of a double argument, which cannot be found in the *Almagest*, is an improvement in computational techniques worked out by Muslim scholars.

After having calculated the effect of lunar parallax by the above-mentioned table, we now need a table for calculating the magnitude and duration of solar and lunar eclipses. This is the *Table of the Solar and Lunar (Hourly) Motion and Diameter of the Shadow and the Interpolation Minutes*. As far as this table is concerned, the *True Records* is highly incomplete, so instead we have to rely upon the *Qizheng tuibu* and the *Huihui lifa* in the *Mingshi*. Concerning the title of the table, more confidence can be placed in the latter. Firstly, for solar eclipses it is necessary to know the apparent diameters of the sun and the moon at the time of the eclipse. These are obtained from the table by taking as the argument the solar anomaly (the distance of the sun from the apogee) in the case of the solar diameter and the lunar epicycle position in the case of the lunar diameter. As has already been mentioned, the mean lunar position at the time of a solar or lunar eclipse always coincides with the apogee. Therefore we need to consider only the change in the apparent lunar diameter caused by the lunar epicycle position. In the table, the apparent solar diameter varies between 32′26″ and 34′48″ depending on the distance from the earth; the apparent lunar diameter varies between 30′30″ and 35′48″. Finally, for lunar eclipses it is necessary to calculate the size of the shadow of the earth, which assumes values between 79′49″ and 98′47″.[22]

The tables discussed above are those needed for the calculation of the eclipses themselves. For knowing the magnitude and duration of an eclipse at the times of sunrise and sunset, the *Table for the Duration of Day and Night* has been added. This table is useful for calculating the arc of daylight and hence the times of sunrise and sunset, but has no direct connection with the actual eclipse calculation. From the table, which has the solar longitude as the argument, the arc of daylight cannot be obtained directly, but is found as the difference of the two tabular values for solar longitudes λ and $\lambda+180°$.[23] Denoting the arc of daylight by $2t$, the declination (orthogonal distance of a point on the ecliptic from the equator) by δ, the geographical latitude of the locality of the observer by ϕ, and the obliquity of the ecliptic by ε, we have

$$\sin\delta(\lambda)=\sin\varepsilon\cdot\sin\lambda \quad \text{and} \quad \cos t(\lambda)=-\tan\phi\cdot\tan\delta(\lambda)\,.$$

When we determine two values of t, for instance from the tabular values for $\lambda=30°$, 90°,

[22] Let us tabulate the modern values of the constants discussed here for comparison:

	maximum value	minimum value
apparent solar diameter	32.6′	31.4′
apparent lunar diameter	33.6′	29.4′
diameter of the shadow of the earth	93.6′	77.0′

[23] The remainder of this paragraph and the following one have been rewritten. (BvD)

we can obtain the unknown value of ϕ using the expressions above. The result is $\phi\approx +32°$, close to the latitude of Nanjing.

Finally, we will discuss the calculation of the numerical values in the *Table for the Duration of Day and Night*. The table can be seen to display the so-called *oblique ascension*, a standard function in Islamic spherical astronomy. If L denotes the point of the ecliptic with longitude λ and P the point of the equator which rises simultaneously with L at a given geographical latitude, then the oblique ascension $\rho(\lambda)$ is the length of the equatorial arc between the vernal equinoctial point and P.[24] The oblique ascension for geographical longitude 0° is called the *right ascension* and will be denoted by $\alpha(\lambda)$; it is the length of the equatorial arc between the vernal equinoctial point and the orthogonal projection of the point on the ecliptic with longitude λ onto the equator. The right ascension for values of the ecliptical longitude in the first quadrant can be found from

$$\tan \alpha(\lambda) = \cos \varepsilon \cdot \tan \lambda .$$

For values in the second to fourth quadrants the right ascension is obtained from the following two symmetry relations:

$$\alpha(180° - \lambda) = 180° - \alpha(\lambda) \quad \text{and} \quad \alpha(180° + \lambda) = 180° + \alpha(\lambda) .$$

Now the oblique ascension for a given value of the geographical latitude can be calculated as $\rho(\lambda) = \alpha(\lambda) - (t(\lambda) - 90°)$. Since $t(180° + \lambda) = 180° - t(\lambda)$, it follows that the difference of the oblique ascension values for arguments $180° + \lambda$ and λ is

$$\{(180 + \alpha(\lambda)) - (180° - t(\lambda) - 90°)\} - \{\alpha(\lambda) - (t(\lambda) - 90°)\} = 2t(\lambda).$$

This result is in exact correspondence with the explanation in the *Huihui li*.

5. Calculation of the Positions of the Five Planets

The calculation of the positions of the five planets as performed by ancient Greek astronomers was for the first time systematized in the *Almagest*. From the beginning, Islamic astronomical tables followed Ptolemy's method and presented only few innovations.

We will first discuss the calculation of planetary longitudes. Although the set-up of the tables for the superior planets Saturn, Jupiter and Mars and the inferior planets Venus and Mercury is completely identical, there are some differences in the method of computation. First we will give concrete numerical values for Saturn as a representative of the superior planets. For both the superior and the inferior planets the geometrical model in the *Almagest* uses a large circle, to be called the *eccentric circle*, from whose centre O the

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[24] A more extensive explanation of the oblique ascension can be found in Otto E. Neugebauer, *A History of Ancient Mathematical Astronomy*, Berlin (Springer) 1975, vol. 1, pp. 30–45. The spherical-astronomical functions tabulated in Islamic astronomical handbooks are summarized on pp. 140–141 of Edward S. Kennedy, "A survey of Islamic astronomical tables", *Transactions of the American Philosophical Society* 46-2 (1956, reprinted in 1989), pp. 123–177. A useful overview of modern spherical astronomy is William M. Smart, *Textbook on spherical astronomy*, revised 6th edition, Cambridge 1977. (BvD)

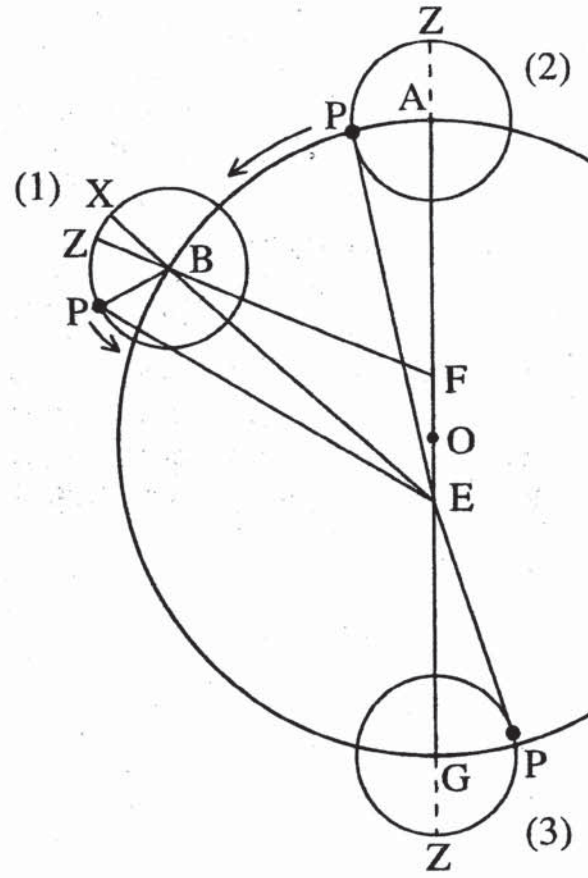

Fig. 3. Calculation of the Longitude of Saturn.

earth E is slightly removed (see Fig. 3). On the opposite side of O a point F, called the *equant*, is taken in such a way that $OE=OF$, and a point B on the eccentric circle is assumed to move uniformly around F. In the *Huihui li* the daily motion of B, which we will indicate by n, is, in the case of Saturn, approximately 2 minutes (more precisely, 2′0″.6).

Now the planet P moves on a small circle around B, the epicycle, in the same direction as the motion of B. In the case of Saturn, the daily motion n' along the epicycle, i.e. the daily motion corresponding to the synodic period, is approximately 57 minutes (57′7″.7). The angle between the reference point Z of the epicycle (on the extension of the line FB) and the planet is called the "own motion[ck]" (mean anomaly). In the *Table of the Apogee Motion and Anomalistic Motion of the Five Planets*[cl] in the *True Records* the values of the mean anomaly are shown together with those for the apogee motion (i.e. the motion of point A) in five tables for collected years, extended years, months, days, and zodiacal signs. Once the mean anomaly of a superior planet is known, its mean position can be obtained by subtracting the mean anomaly from the mean solar position. This follows from the fact that in the *Almagest* model for the superior planets the sun is always positioned in the direction of $\overrightarrow{BP}$. For the inferior planets, on the other hand, the sun is always positioned in the direction of $\overrightarrow{FB}$, so that the mean position of the planet is equal to that of the sun, whereas the anomalistic motion, i.e. the motion on the epicycle, takes place with the synodic period. In either case, after the mean position has been found like this, we can obtain the "position of the centre of the epicycle[cm]" (mean centrum) $\angle AFB$ by subtracting

Table 1. The Daily Mean Motion *n* and Mean Anomalistic Motion *n′* of the Five Planets.

planet	daily motions		remarks
	n	*n′*	
Saturn Jupiter Mars	2′ 0″.6 4′59″.2 31′26″.6	57′ 7″.7 54′ 9″.1 27′41″.8	*n*+*n′* equals the daily solar motion 59′8″.3
Venus Mercury	59′ 8″.3 59′ 8″.3	37′ 3° 6′	*n* equals the daily solar motion

the apogee position, which can also be calculated from the tables. The values of *n* and *n′* for the five planets are shown in Table 1.

Following the method of the *Almagest*, a conversion is performed from the point *Z* (in the Almagest called the "mean apogee of the epicycle") as the starting point of the anomalistic motion to the point *X* on the extension of line *EB*. Thus we obtain ∠*XBZ*= ∠*EBF*, which can be used not only for the correction of the anomaly but also to calculate ∠*AEB* from ∠*AFB*. The values of this correction, called the "first equation[cn]", are given in the *Table of the First Equation and Interpolation Minutes*[co] with the position of the epicycle centre obtained above as the argument. The first order differences of the equation are called "equation minutes[cp]". We can calculate the constants of the model in Fig. 3 by working backwards from the tabular values. When we take the radius of the eccentric circle equal to 1, the value of *EF* for Saturn becomes 0.110. As in the case of the moon, the mean position of the epicycle centre corrected by the first equation is ∠*AEB*, the "true position of the epicycle centre[cq]", and in the same way we obtain the "true anomaly[cr]" ∠*XBP* by correcting the mean anomaly.

In the *True Records*, the third and last part of the Outer Book is further divided into three chapters, the first of which contains the above-mentioned *Table of the First Equation and Interpolation Minutes*. In the second chapter we find the T*able of the Second Equation and Distance Degrees*[cs], from which ∠*BEP* in Fig. 3 can be found as a function of the true anomaly. This table has been made on the basis of precisely the same considerations as the similar table for the moon. Thus, values of ∠*BEP* have been calculated as a function of the true anomaly for the cases that the centre *B* of the epicycle coincides with *A* or with *G*. The values for *A* are those of the "second equation[ct]", the differences in ∠*BEP* between situations *G* and *A* for the same value of the true anomaly are the "distance degrees[cu]". Again the distance degrees are multiplied by the "interpolation minutes[cv]", which are obtained from the preceding *Table of the First Equation and Interpolation Minutes* with the mean position of the epicycle centre as the argument. The result, divided by 60, is added to the second equation to obtain the "definite equation[cw]". By adding the definite equation obtained in this way to or subtracting it from the true position of the epicycle centre and then adding the result to the apogee position, the planetary longitude can be found.

We will now consider our example Saturn at a time when the epicycle centre is in

Table 2. The Dimensions of the planetary orbits in the *Huihui li* and the *Almagest*. All values are given under the assumption that the radius of the eccentric circle Equals 1, although in the *Almagest* this radius was taken equal to 60.

		Saturn	Jupiter	Mars	Venus	Mercury
Radius of the epicycle	*Huihui li*	0.1042	0.1882	0.6582	0.7198	0.3490
	Almagest	0.1083	0.1917	0.6583	0.7194	0.3750
Eccentricity (*EF*)	*Huihui li*	0.110	0.0889	0.2003	0.0357	0.0483
	Almagest	0.114	0.0916	0.2000	0.0417	0.1000

point *A* and the planet reaches a point *P* such that the line *EP* from the earth to the planet is tangent to the epicycle (situation (2) in Fig. 3). Taking the radius of the eccentric circle equal to 1, the value of *EA* is approximately 1.055. Furthermore, $\angle AEP$ reaches its maximum value 5°40′ (taken from the table) for a true anomaly in the neighbourhood of 96 degrees, from which we can derive the value 0.1042 for the radius of the epicycle. Since the distance *EG* is 0.945, we can calculate the maximum of $\angle PEG$ to be approximately 6°20′ (situation (3) in Fig. 3). This is in good agreement with the maximum value 6°21′ of the sum of the second equation and the distance degrees which we find in the table for true anomaly values around 96 and 243 degrees. In Table 2 we have brought together the constants for all five planets calculated from the tables according to this same method.[25] By the way, by a direct comparison it can easily be seen that the tables for the first and second equations discussed above are much simpler than those in the *Almagest*. Also here we can recognize the efforts of Muslim astronomers.

For the present we omit the *Table of Progressions and Stations*[cx] recorded in the second chapter of the third part of the Outer Book of the *True Records* and will touch on the *Table of the Ecliptical Latitude of the Five Planets*[cy] in the third and last chapter. Whereas in the calculation of the planetary longitudes it was assumed that the eccentric circle and the epicycle both lie in the plane of the ecliptic, for the calculation of the latitudes we now presume that they are inclined to it. However, since the angles of inclination are

Table 3. The Constants of the models for the planetary orbits according to Itaru Imai. In this table the distance of the apogee from the earth has been taken as the unit.

	Saturn	Jupiter	Mars	Venus	Mercury
Radius of the eccentric circle	0.9464	0.9572	0.9091	0.9825	0.9233
Radius of the epicycle	0.0987	0.1802	0.5983	0.7067	0.3409
Distance of the apogee	1.0000	1.0000	1.0000	1.0000	1.0000
Distance of the perigee	0.8928	0.9145	0.8181	0.9650	0.8465
Eccentricity	0.0566	0.0447	0.1001	0.0178	0.0832

[25] According to calculations by Imai Itaru, who took the distance of the apogee from the earth as the unit, the constants of the models are as shown in Table 3; see Imai Itaru 今井溱, "The planetary motions in the Ming translation of the *Huihui li*" (明訳回回暦書の惑星運動, in Japanese), Tenkansho 天官書 9 (private publication).

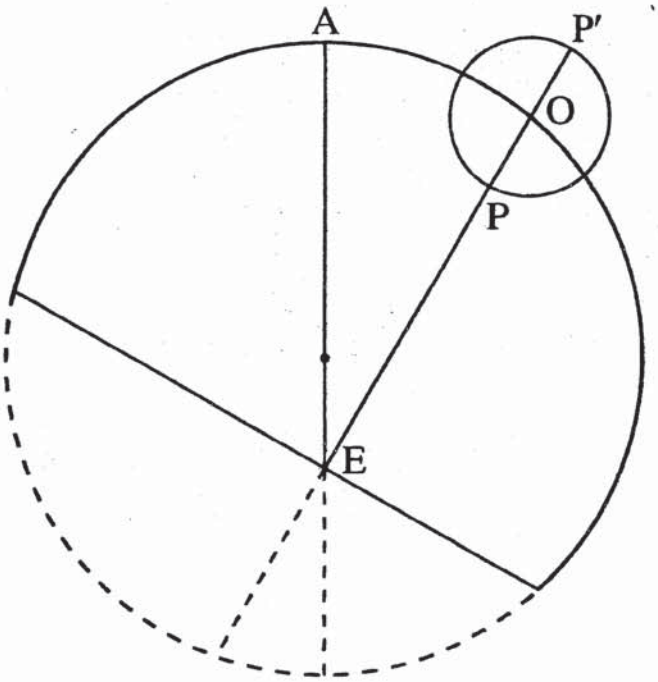

Fig. 4. The Latitude Model for Saturn (view from above).

small, this is considered not to have any influence on the longitude values. We will first discuss the latitude of the superior planets, again taking Saturn as example.

The latitude tables for all five planets have the "definite anomaly[cz]" and the "definite position of the epicycle centre[da]" as a double argument.[26] Furthermore, the tables were as much as possible simplified by letting 120 degrees of the definite anomaly and 60 degrees of the definite position of the epicycle centre correspond to a complete rotation. Accordingly, to convert the definite anomaly and the definite position of the epicycle centre in the tables into the actual values, they must be multiplied by three and six respectively. At any rate, these devices are peculiar to Islamic astronomical tables and a large improvement over the tables in the *Almagest*.

We will now discuss the inclination of the eccentric circle with respect to the ecliptic. For the superior planets the eccentric circle lies in a fixed plane inclined to the ecliptic by an angle I. In the case of Saturn the northernmost point of the eccentric circle, which we will call the *northern limit*, does not coincide with the apogee. According to the table it occurs for a position of the epicycle centre (measured from the apogee) equal to 300°, whereas the *southern limit* (the southernmost point of the eccentric circle) is found symmetrically opposite this point, i.e. for a position of the epicycle centre equal to 120°. When we take the radius of the eccentric circle equal to 1, the following values can be calculated for the northern limit. Let O be the centre of the epicycle, P the point of the epicycle closest to the earth, and P' the point of the epicycle farthest from the earth (Fig. 4). Then we have $EO = 1.026$, $EP' = 1.130$, and $EP = 0.922$.

[26] Although these two variables have the same Chinese names as the true anomaly and true position of the epicycle centre found above, they are in fact scaled versions of the *mean* anomaly and the *mean* position of the epicycle centre. (BvD)

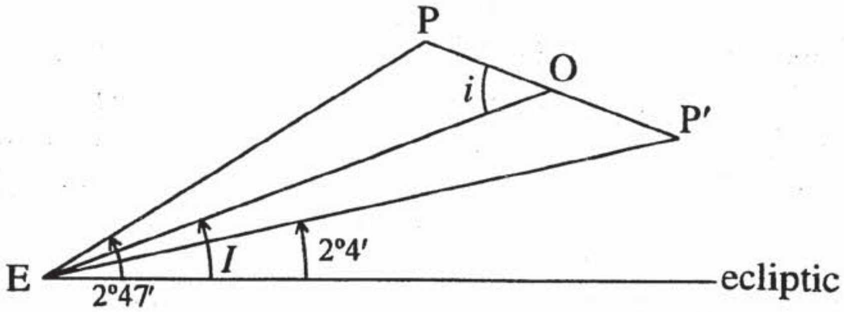

Fig. 5. The Latitude Model for Saturn (side view).

From the tables we find that the latitude of Saturn at its northern limit (i.e. for a position of the epicycle centre equal to 300°) is 2°47′ when the planet is in P (anomaly equal to 180°) and 2°4′ when the planet is in P' (anomaly equal to 0°). At the northern limit, O is removed from the ecliptic by an angle I. If we furthermore assume that the epicycle is inclined to the eccentric circle by an angle i, the values of I and i can be calculated from the latitude values given above using Fig. 5. The results being $I=2°23'$ and $i=3°30'$, it follows that the inclination of the epicycle with respect to the ecliptic, the angle $i-I$, is only 1°7′, which means that the epicycle is almost parallel to the ecliptic. The constants for Jupiter and Mars, calculated in the same way, are listed in Table 4. For Jupiter the northern limit coincides with the apogee of the eccentric circle, for Mars it differs only slightly. Both for Jupiter and for Mars the epicycle is practically parallel to the ecliptic.

In the same way as in the *Almagest*, the geometrical model used in the *Huihui li* for the calculation of the latitudes of Venus and Mercury is more complicated than that for the superior planets. In particular, it can be noted that for the inferior planets the inclination of the epicycle with respect to the eccentric circle changes continuously as a function of the position of the epicycle centre in two different ways. Firstly, in the direction of the apogee A of the eccentric circle the diameter HK of the epicycle perpendicular to EA is inclined to the eccentric circle (see situation (1) in Fig. 6; the solid part of the epicycle is north of the eccentric circle, the dotted part south). In the perigee B the diameter HK is inclined to the eccentric circle by the same angle but in the opposite direction (situation (3) in Fig. 6). We will call this inclination the *slant*. It reaches its maximum value j when the centre of the epicycle is in A or B; in between these two points the slant gradually decreases, becoming zero in the direction of the line NEN perpendicular to AB. Secondly, the inclination with respect to the eccentric circle of the diameter LM of the epicycle is zero when the centre of the epicycle is in A or B; from there on it gradually increases and

Table 4. Constants of the Latitude Models for the Superior Planets.

	Saturn	Jupiter	Mars
Northern limit (reckoned from the apogee)	300°	0°	354°
Inclination of the eccentric circle (I)	2°23′	1°16′	1°22′
Inclination of the epicycle ($i-I$)	1° 7′	18′	39′

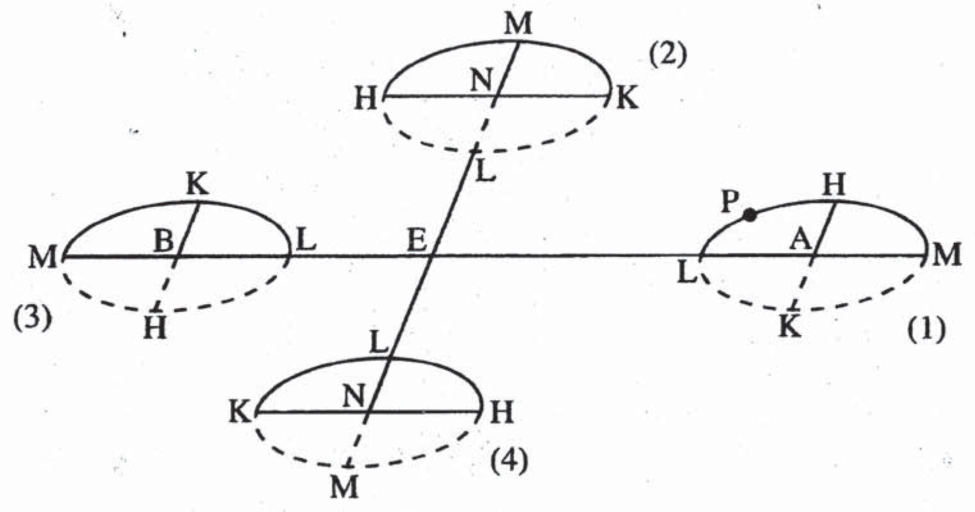

Fig. 6. The Latitude Models for Venus and Mercury (view from above).

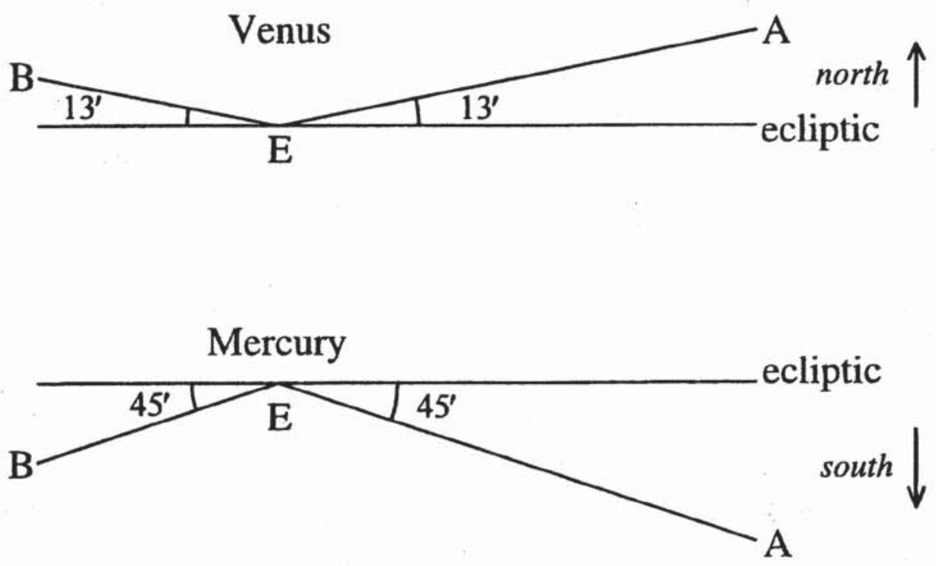

Fig. 7. The Latitude models for Venus and Mercury (side view).

reaches its maximum value when the centre of the epicycle is in N (situations (2) and (4) in Fig. 6). This inclination we will call the *deviation* and we will denote its maximum by J.

[27]In the case of the inferior planets not only the slant and the deviation change continuously, but also the inclination of the eccentric circle is variable. Both for Venus and for Mercury the inclination is zero when the centre of the epicycle reaches N, and it assumes its largest value when the centre of the epicycle is in A or B. As a result, the centre of the epicycle will always be on the same side of the ecliptic (Fig. 7); in the case of Venus it is always north of the ecliptic, in the case of Mercury always south. The maximum inclinations can easily be found, since they are the respective latitudes when the centre of the epicycle is in A or B (i.e. the position of the centre of the epicycle is 0° or

[27] This paragraph has been rewritten. (BvD)

180°) and the anomaly is 0° or 180°. For Venus the maximum is 13′, for Mercury 45′.

The inclination of the eccentric circle as well as the deviation and slant of the continuously changing epicycle described above are reflected in the numerical values of the latitude tables. Therefore, let us calculate the values of the maximum deviation J and the maximum slant j by working backwards from the tabular values. First we will discuss the slant in the case of Venus. We have already mentioned that the slant reaches its maximum value in the apogee of the eccentric circle (for a position of the epicycle centre equal to 0°) and in its perigee (180°). Furthermore, it reaches this maximum value in the direction of *HK*, i.e. when the anomaly is 90° or 270°. In the table we find the following latitude values:

Position of the epicycle centre	Anomaly	
	90°	270°
0° (apogee)	1°58′ north	1°32′ south
180° (perigee)	1°32′ south	1°58′ north

Thus the values for these four situations are pairwise identical. This indicates that in the calculation of the latitudes the distance of the epicycle centre from the earth can be regarded to be the same in each case. A similar assumption is valid in the case of Mercury. Since both for Venus and for Mercury the eccentricity *EF* (see Table 2) is small, only a minor error results if the latitudes are calculated for a fixed distance between the earth and the centre of the epicycle. It can probably be assumed that for this fixed distance the radius of the eccentric circle was used. Moreover, if we disregard the direction (north or south) of the latitude, the average of the values for Venus quoted above is 1°45′, differing 13′ from the northern and southern values. This difference is the maximum inclination of the eccentric circle for Venus, whose value has already been mentioned. Furthermore, it can be seen that precisely the 1°45′ is the value of the latitude produced by the slant. Once we know this, the maximum slant j is easily calculated. In the case of Mercury a completely identical argument can be applied.

Finally, the deviation of the epicycle assumes its maximum value J in the direction *NN* (situations (2) and (4) in Fig. 6). When in this case Venus reaches L its latitude is 7°15′, south of the ecliptic if the position of the epicycle centre is 90°, north if it is 270°. Using these values the maximum deviation J can be calculated. The results obtained for both Venus and Mercury have been brought together in Table 5.

Table 5. Constants of the Latitude Models for Venus and Mercury.

	Maximum inclination I	Maximum slant j	Maximum deviation J
Venus	0°13′	3° 0′	2°49′
Mercury	0°45′	6°23′	7°36′

6. Examination of the Star Table

As we have seen, the Islamic astronomy transmitted to China has been recorded in three works: the *Qizheng tuibu*, the *True Records of King Sejong* of the Korean Yi Dynasty, and the *Huihui lifa* in the *Mingshi*. Only the first two works contain a detailed table of fixed stars, which gives the ecliptical longitudes and latitudes of 278 stars around the zodiac. The names of the stars are listed by an ordinal number within the twelve zodiacal signs, in contrast to the traditional Chinese names. Nearly all star tables in Islamic astronomy are based on Ptolemy's star catalogue in the *Almagest* and indeed the numbering of the stars in the *Qizheng tuibu* and the *True Records* clearly agrees with Ptolemy's order.

In his investigation of Ptolemy's star catalogue, Knobel states that until the new observations of the fixed stars by Ulugh Beg in Samarkand around the middle of the fifteenth century, Islamic astronomers simply adapted the longitudes in Ptolemy's star catalogue to their own era by performing a correction for precession, while leaving the latitudes unchanged.[28] However, since the latitude values found in the almost identical star tables in the *Qizheng tuibu* and the *True Records* are different from Ptolemy's, it can for the present be presumed that they derive from entirely new observations. Furthermore, whereas none of the Islamic star tables contains stars which are not among the 1022 recorded by Ptolemy, the star table in the *Qizheng tuibu* and the *True Records* includes a small number of completely new stars with the indication "newly interpreted star not belonging to a constellation[db]".[29] This means that an investigation of this star table will provide the study of Islamic astronomy with entirely new material.

An investigation of the star table in the *Qizheng tuibu* and the *True Records* from the above perspective has been published by the present author on earlier occasions.[30] From a comparison with Ptolemy's star catalogue a rough identification of the stars was obtained. Then from the brightest stars recorded four were chosen both from the first and from the second category and the years corresponding to their ecliptical longitudes were determined. As is shown in Table 6, the resulting average year of observation was 1365, which is three years before the beginning of the reign of Hong Wu. Since the longitude values naturally contain observational errors, this year is not definitely fixed. In the English publication by the present author in the *Silver Jubilee Volume* (see footnote 30) the identifica-

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[28] Christian H. F. Peters & Edward B. Knobel, *Ptolemy's Catalogue of Stars, a Revision of the Almagest*, Washington 1915. Concerning Islamic star tables, see also: Paul Kunitzsch, "Star Catalogues and Star Tables in Mediaeval Oriental and European Astronomy", *Indian Journal of History of Science* 21 (1986), pp. 113–122 (reprinted as Chapter I in P. Kunitzsch, *The Arabs and the Stars: Texts and Traditions on the Fixed Stars, and their Influence in Medieval Europe*, Northampton (Variorum Reprints) 1989).

[29] Whereas since Ptolemy stars were assigned an ordinal number within the twelve zodiacal signs, whose names were those of constellations, the names of the zodiacal signs are missing in the case of the "newly interpreted stars not belonging to a constellation". Chapter 3, *Categories of calendar calculation* (曆算類), of the *New compilation of the annals of literature from the Yuanshi* (元史新編藝文志) contains a section *Newly observed stars without name* (新測無名諸星). The relationship between this section and the new stars in the *Qizheng tuibu* is unclear.

[30] Whereas a brief report can be found in "Islamic astronomy in China" (中国に於けるイスラム天文學, in Japanese), *Tōhō Gakuhō* 東方學報 19 (1950), pp. 65–75, a detailed discussion is contained in "Indian and Arabian Astronomy in China", in *Silver Jubilee Volume of the Zinbun Kagaku Kenkyusyo*, Kyoto 1954.

Table 6. Determination of the Year of Observation of the Star Table in the *Qizheng tuibu*.

Stars of the constellations north and south of the ecliptic		Modern name	Recorded longitude	Estimated year of observation
Inside Taurus	no. 14	α Tau	60°51′	1360
Inside Gemini	no. 1	α Gem	101°33′	1376
Inside Gemini	no. 2	β Gem	104°44′	1384
Inside Leo	no. 6	γ Leo	140°34′	1356
Inside Leo	no. 8	α Leo	140°52′	1355
Inside Virgo	no. 14	α Vir	194°40′	1342
Inside Libra	no. 2	δ Sco	233°54′	1378
Inside Libra	no. 8	α Sco	240°59′	1370
		Average year		1365

tion of all 278 stars and a comparison of all recorded longitudes with computed ones were carried out, tentatively taking the year of observation as 1365. Only small differences between text and computation could be noticed.

At the time when the above research was performed, the present author had in fact not yet seen the *True Records of King Sejong*. In that work the following double-column notes are found under the title of the *Table of the Longitudes and Latitudes of the Stars Inside and Outside of each Constellation North and South of the Ecliptic*[dc]:

> "To the longitude of each constellation 4 minutes are added in each 5 years. In the year *bing-zi* of Hong Wu (1396) the accumulated years amount to 798, so already 4 minutes have been added. When we come to the year *xin-si* (1401), they amount to 803, so naturally 4 minutes should be added again. Adding these repeatedly for (every) 5 years, we come to eternity."

From this it is seen that the longitudes in the star table are not observed values as such, but were corrected for precession by increasing them by four minutes in every five years, i.e. by 48 seconds per year. Since for the values for the year 29 of Hong Wu (the year *bing-zi*, 1396) "already 4 minutes had been added", it can be conjectured that the observations took place five years earlier, i.e. in the year 24 of Hong Wu (1391). The difference between this year and 1365 is 26 years, but if we accept an error level of 20 minutes in the observed longitude values, this result is by no means incompatible with the date found before.

Because of the disagreement with Ptolemy's star catalogue as far as the ecliptical latitudes are concerned, the star table in the *Qizheng tuibu* and the *True Records* should be considered to be the result of new observations at the end of the fourteenth century. Whether these observations were carried out in China, or the *Huihui li* reproduces an Islamic star table which had been transmitted to China after the introduction of the dust-board methods in the year 18 of Hong Wu, can not easily be decided.

7. Conclusion

In the above we have examined the main part of the astronomical tables found in the *Huihui li* and have also briefly discussed its star table. The *Huihui li* essentially follows ancient Greek astronomy and the numerical values in its tables were calculated on the basis of the geometrical planetary models in Ptolemy's *Almagest*. From our examination of the tabular values we found that, for instance in the case of the solar eccentricity, the *Huihui li* uses values for the astronomical constants which are superior to those in the *Almagest*. At the same time particularly much attention has been paid to the structure of the tables for calculating solar eclipses and planetary positions, which have a double argument and hence are extremely simple to use. Of course, the improvement of the values of the astronomical constants and the simplification of the use of the tables may not have originated in the Chinese translation of the *Huihui li*; such improvements can be assumed to have been made step by step all through the history of Islamic astronomy. The examination of this problem, as well as the determination of the original source of the *Huihui li*, require a detailed investigation of Islamic astronomy itself. Finally, the star table recorded in the *Qizheng tuibu* and the *True Records* has been studied, but also here various problems remain.

Appendix (by the translator)

This appendix first lists as alphabetically numbered notes to the main text the Chinese characters for book titles and names of technical concepts which have been translated into English. Then follow short descriptions of the most important persons and works discussed in this article and finally a short explanation of the method by which years were counted in Chinese historical sources.

Chinese characters

[a]輟耕録 [b]麻答把曆 [c]崇禎曆書 [d]司屬司天監 [e]元秘書監志 [f]西域儀象 [g]咱禿哈刺吉 [h]混天儀 [i]咱禿朔八台 [j]測驗周天星曜之器 [k]魯哈麻亦渺凹只 [l]春秋分晷影堂 [m]魯哈麻亦木思塔餘 [n]冬夏至晷影堂 [o]苦來亦撒麻 [p]渾天圖 [q]苦來亦阿兒子 [r]地理志 [s]兀速都兒刺 [t]晝夜時刻之器 [u]都水少監 [v]觀星臺 [w]元朝名巨事略 [x]簡儀 [y]高表 [z]候極儀 [aa]渾天象 [ab]玲瓏儀 [ac]仰儀 [ad]立運儀 [ae]證理儀 [af]景符 [ag]闚几 [ah]日月食儀 [ai]星晷 [aj]定時儀 [ak]正方案 [al]大明殿燈漏 [am]圭表 [an]仰釜日晷 [ao]革象新書 [ap]欽定四庫全書提要 [aq]西域阿刺必年 [ar]疇人傳 [as]法而斡而丁 [at]阿而的必喜世 [au]虎而達 [av]提而 [aw]木而達 [ax]沙合列斡而 [ay]列(別?)黒而 [az]阿斑 [ba]阿咱自 [bb]答亦 [bc]八哈慢 [bd]亦思番達而麻的 [be]求宮分閏日 [bf]求月分閏日 [bg]求中國閏月 [bh]西域歳前積年 [bi]最高行度 [bj]日中心行度 or 日中行度 [bk]自行度 [bl]太陽加減差 [bm]太陽最高行度立成 [bn]日中行度立成 [bo]太陽加減差分立成 [bp]加減分(corrected from 加減差分，BvD) [bq]中心行度 [br]加倍相離度 [bs]本輪行度 [bt]第一加減差 [bu]本輪行定度 [bv]比敷分 [bw]第二加減差及遠近度立成 [bx]第二加減差 [by]遠近度 [bz]加減定差 [ca]經度 [cb]羅計中心行度立成 [cc]計都與月相離行度 [cd]太陰黄道南北緯度立成 [ce]晝夜加減差立成 [cf]經緯時加減差立成

[cg]太陽太陰行影徑分及比敷分立成 [ch]晝夜時宮度分立成 [ci]食甚泛時 [cj]求子正至合朔時分秒 [ck]自行度 [cl]五星最高行度及自行度立成 [cm]小輪心度 [cn]第一加減差 [co]第一加減差比敷分立成 [cp]加減分（corrected from 加減差分，BvD） [cq]小輪心定度 [cr]自行定度 [cs]第二加減遠近立成 [ct]第二加減差 [cu]遠近度 [cv]比敷分 [cw]加減定差 [cx]順留立成 [cy]五星黄道南北緯度立成 [cz]自行定度 [da]小輪心定度 [db]新譯星無像 [dc]黄道南北各像内外星経緯度立成

Persons

Bei Lin 貝琳 Vice-director of the Astronomical Bureau during the Ming Dynasty. Prepared a reworking of the *Huihui li*, entitled *Qizheng tuibu*, in the year 1477.

Chinggis Khan（太祖 Tai Zu, 1162–1227） Founder of the Mongol world empire. Was proclaimed khan of all Mongols in 1206.

Guo Shoujing 郭守敬 Excellent engineer, mathematician and astronomer from the second half of the thirteenth century. One of the authors of the *Shoushi li* and builder of a large number of astronomical instruments.

Hülegü Khan Younger brother of Khubilai Khan. As the first Il-Khan (1254–1265) he ordered the construction of the astronomical observatory in his capital Maragha in north-western Iran.

Khubilai Khan（世祖 Shi Zu） Grandson of Chinggis Khan. First emperor of the Yuan Dynasty (1260–1294).

Möngke（憲宗 Xian Zong） Oldest brother of Khubilai Khan. Third Mongol Great Khan (1251–1259).

Naṣīr al-Dīn al-Ṭūsī Persian astronomer. Director of the observatory in Maragha, which he built on the order of Hülegü Khan. Author of the *Īlkhānī Zīj*.

Ögödei（太宗 Tai Zong） Son and successor of Chinggis Khan (1229–1241).

Yelü Chucai 耶律楚材 (1190–1244) Chinese astronomer and high official in the service of Chinggis Khan.

Zhamaluding 札馬魯丁 (Jamāl al-Dīn) Muslim astronomer who came to China in the early Yuan dynasty and presented seven instruments of western origin. He compiled the *Wannian li* and was the first director of the Islamic Astronomical Bureau.

Calendars and other primary sources

True Records of King Sejong（世祖實録 *Sejong Sillok*） Collection of official records of the Korean Yi Dynasty (1393–1910). A reworking of the *Huihui li* by Korean astronomers is contained in the volume *Chiljongsan* of this work.

Daming li 大明曆 Calendar of which a revised version was in use by the Jin Dynasty. It was adopted by Chinggis Khan after his conquest of the Jin capital Zhongdu (Beijing) in 1215.

Datong li 大統曆 Calendar of the Ming Dynasty, basically identical to the *Shoushi li*.

Huihui li 回回曆 "Islamic calendar", an astronomical handbook with tables of Islamic type which probably originated in the early Yuan Dynasty. The original text is lost, but three Chinese versions are contained in the *Mingshi*, the *Qizheng tuibu*, and the *True Records of King Sejong*.

Mingshi 明史 Official annals of the Ming Dynasty.

Qizheng tuibu 七政推步 Reworked version of the *Huihui li* by Bei Lin (1477).

Shoushi li 授時曆 Famous calendar of the Yuan Dynasty, compiled by Guo Shoujing and others.

Tongtian li 統天曆 Calendar of the Southern Song Dynasty.

Wannian li 萬年曆 Calendar of Islamic type presented to the first Yuan emperor Khubilai Khan by the Muslim astronomer Zhamaluding (1267).

Western Expedition Calendar with Epoch Year Geng-Wu (西征庚午元曆 *Xi-zheng geng-wu yuan li*) Calendar compiled by Yelü Chucai. It was an adaptation of the *Daming li* to the geographical longitude of Samarkand, residence of Chinggis Khan in the year 1220.

Yuanshi 元史 Official annals of the Yuan Dynasty.

Chinese year count

In Chinese historical sources the years are counted in two ways: from the beginning of the current reign period and/or using the sexagesimal cycle. The reign periods occurring in this article, listed with the respective dynasties, are as follows (note that till the Yuan dynasty many emperors used more than one different reign period):

Dynasties			*Reign periods*		
隋	Sui	581–618	開皇	Kai Huang	581–601
唐	Tang	618–906	武德	Wu De	618–627
金	Jin	1115–1260	天會	Tian Hui	1123–1138
南宋	Southern Song	1127–1280			
元	Yuan	1260–1368	至元	Zhi Yuan	1264–1295
明	Ming	1368–1644	洪武	Hong Wu	1368–1399
			成化	Cheng Hua	1465–1488
			崇禎	Chong Zhen	1628–1644
清	Qing	1644–1912			

The names of the elements of the sexagesimal cycle are built up from a decimal and a duodecimal component, reason why the names of the seventh and seventeenth elements below have the first component in common and those for the first and thirteenth elements the second one.

1	甲子 *jia-zi*	13	丙子 *bing-zi*	18	辛巳 *xin-si*	56	己未 *ji-wei*
7	庚午 *geng-wu*	17	庚辰 *geng-chen*	19	壬午 *ren-wu*		

Epilogue

Since Professor Yabuuti originally wrote the above chapter on Islamic astronomy in the Yuan and Ming Dynasties, a number of new studies on the *Huihui li* have been conducted. At the Institute for the History of Natural Sciences of the Academia Sinica in Beijing, Professor Chen Meidong 陈美东 carried out an analysis of the mean motion parameters in the *Huihui li*, whereas Professor Chen Jiujin 陈久金 and his student Yang Yi 杨怡 prepared descriptions of the underlying theory of the solar, lunar and planetary models. Most of the results were published in *Studies in the History of Natural Sciences* (自然科学史研究, Chinese with English summaries).[31]

In a research project at Kyoto Sangyo University, financed by the Japan Society for the Promotion of Science (日本学術振興会) during the period from September 1995 till August 1997, a renewed analysis of mathematical aspects of the *Huihui li*, as well as a comparison of its contents with Islamic astronomical works written in Arabic and Persian has been carried out by the undersigned. Taking the planetary latitude tables as a case study, it was found that the *Huihui li* was highly innovative both in its values of the underlying planetary parameters and in the structure of its tables. The only *zīj* (Islamic astronomical handbook with tables and explanatory text) which turned out to be directly related to the *Huihui li* was written by a certain al-Sanjufīnī in the year 1366 and is extant in the unique manuscript Paris Bibliothèque National Arabe 6040. Al-Sanjufīnī originated from the region of Samarkand, but wrote his *zīj* in Arabic on the order of the Mongol vice-roy of Tibet, a direct descendent of Chinggis Khan. The *Huihui li* and the so-called Sanjufīnī *Zīj* have a number of tables in common, viz. those for the oblique ascension, the equation of time, the lunar latitude, and the lunar and planetary equations. Furthermore, the Sanjufīnī *Zīj* contains a standard set of tables for planetary latitude, from which the sophisticated tables in the *Huihui li* seem to have been derived. Further research is necessary in order to establish whether the *zīj* of al-Sanjufīnī is the original Islamic work which was "presented by a foreigner who became naturalized in China" and was then translated into Chinese (cf. Section 2 above), or simply contains material from an Arabic or Persian version of the *Huihui li* compiled in the early Yuan Dynasty, of which now only the modified Ming translation is extant in the sources described by Professor Yabuuti.[32]

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[31] See, in particular, vol. 5 (1986), pp. 11–21; vol. 8 (1989), pp. 219–229; vol. 9 (1990), pp. 119–131; and vol. 10 (1991), pp. 246–258. Additional articles on Islamic astronomy in China can be found in the *Collected Works of Chen Jiujin* (陈久金集, in Chinese), Harbin 1993. See also: Chen Meidong, "A Study of Some Astronomical Data in Muslim Calendar", in *History of Oriental Astronomy: Proceedings of an International Astronomical Union Colloquium No. 91 (New Delhi, India, 13–16 November 1985)*, Cambridge 1987, pp. 169–174, and Chen Jiujin, "Comparative Research between the Huihui Calendar, Chiljŏngsan Oepiŏn and Qizheng Tuibu", in *Oriental Astronomy from Guo Shoujing to King Sejong: Proceedings of an International Conference, Seoul 6–11 October 1993*, Seoul 1997, pp.105–111.

[32] Some preliminary results of the above research will be published as: Michio YANO and Benno VAN DALEN, "Tables of Planetary Latitude in the *Huihui li*—Parts I and II", in *Proceedings of the 8th International Conference on the History of Science in East Asia, Seoul (Korea), 26–31 August 1996.*