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Tables of Planetary Latitude in the *Huihui li* (I)*

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Professor Kiyosi Yabuuti was the first historian of astronomy who undertook scholarly surveys on the *Huihui li* 回回曆, a Chinese text with astronomical tables compiled at the Islamic Bureau of Astronomy in the Yuan-Ming period. It was because he was well aware of the crucial role of the Ptolemaic planetary system in the *Huihui li* that he read Halma's French translation of Ptolemy's *Almagest* and eventually translated it into Japanese. Without his pioneering work[1] our study would have been far more difficult.

Although Yabuuti has clarified almost all the topics in the *Huihui li* by comparing them with those in the *Almagest*, he left some room for more detailed analyses such as these attempted in our present research. An especially interesting and important problem is the search for a possible source of the *Huihui li* and a reconstruction of the original form of the Chinese translation which must have been made during the Yuan Dynasty. For this purpose we should be informed of Arabic and Persian texts on astronomy, especially those belonging to

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1 Originally published as "Kaikaireki kai" 回回曆解, *Tōhōgakuhō* 東方學報 36 (Kyoto, 1964), 611-32. Almost the same contents are found in his *Chūgoku no tenmon rekihō* 中國の天文曆法, pp. 215-34. See also K. Yabuuti, "The Influence of Islamic Astronomy in China," in D. King and G.A. Saliba (eds.), *From Deferent to Equant: A Volume of Studies in the History of Science in the Ancient and Medieval Near East in Honor of E.S. Kennedy. Annals of the New York Academy of Science*, vol. 500, pp. 547-59. Recently Chen Jiujin 陳久金 published a paper on the *Huihui li*: "Huili riyue weizhi di jisuan ji yundong di jihe muxing" 回曆日月位置的計算及運動的幾何模型 (The Calculation of Positions of the Sun and the Moon and Geometric Models of Their Motion in Hui Calendar), *Ziran kexueshi yanjiu* 自然科學史研究 (Studies in the History of Natural Sciences) 8(3) (1989), 219-29. For contacts between Chinese and Islamic exact sciences, see Jean-Claude Martzloff's brief summary in *A History of Chinese Mathematics* (Springer, 1997), pp. 101-105.

the category called *Zīj* (astronomical handbook with tables). No one before us, however, has taken the trouble of reading such sources and comparing them with the *Huihui li*. One of the reasons is that very few printed editions of *Zījes* are available, so that almost all the information we can expect is still in the form of manuscripts. The present study on the latitude tables offers an example of our new approach to the *Huihui li* which uses information from *Zījes*.

There are three different recensions of the *Huihui li*, namely, (1) that recorded in the official *Ming Dynastic History* (*Mingshi* 明史),[2] which was compiled during the Qing Dynasty, (2) the *Qizheng tuibu* 七政推步[3] compiled by Bei Lin 貝琳 in 1477, and (3) the Korean recension *Chilchŏng san* 七政算 which forms a part of the *Sejong sillok* 世宗實錄 compiled during the reign of King Sejong (r. 1419–50).[4] These recensions are considerably different, especially in the arrangement and order of the explanatory texts and tables.

The Latitude of the Five Planets

Determining the latitude of the planets was one of the most difficult problems in ancient astronomy. The difficulty was inevitable in the geocentric cosmology where the plane of a planet's orbit cannot have a fixed inclination to the plane of the ecliptic. This is why Ptolemy put the theory of planetary latitude in the last Book of the *Almagest*. In ancient Chinese astronomy, where no geometrical model was conceived, the problem should have been still more difficult, as is witnessed by the absence of a systematic discussion of this problem before the Yuan Dynasty.

In the introduction to the *Huihui li*, we read:

> In the autumn of the year 15 (A.D. 1382) emperor Taizu said: "The Western investigations of the heavenly phenomena are most refined. Moreover, their (theory of the) latitudes of the five planets is not available in China."[5]

Similar words are found[6] in the *Mingyi tianwen shu* 明譯天文書, which

2 We have used *Mingshi* 明史 "Lizhi" 曆志 contained in *Lidai tianwen lüli dengzhi huipian* 歷代天文律曆等志彙編 10.

3 We have used the edition of the *Qizheng tuibu* contained in *Jinding siku quanshu* 欽定四庫全書, zibu 子部 6.

4 We have used the reprint edition of the *Sejong sillok* published by the Oriental Institute of Gakushūin 學習院 University.

5 十五年秋, 太祖謂西域推測天象最精, 其五星緯度又中國所無.

6 爾來西域陰陽家, 推測天象, 至爲精密有驗. 其緯度之法, 又中國之所未備.

is a Chinese translation of Kūšyār ibn Labbān's book on astrology[7] completed in the same year by the same group of scholars.

The Chinese text on planetary latitude, in all three recensions, is very brief, just explaining how to use the tables. The tables, however, are extensive. It seems that the translators were less interested in the theoretical exposition of the problem than in the practical use of the tables.

Translation and Commentary

In what follows we shall give a translation of the Chinese text on planetary latitudes recorded in the *Ming Dynastic History*, vol. 37. The sentences in smaller point are commentaries in the original text, and the words within brackets are our additions. We have attached the Chinese text as Appendix 1. The tables of planetary latitude occupy a large part of vol. 39 of the *Ming Dynastic History*. We have shown an example of Saturn's table of latitude in Appendix 2. The original format is shown in Appendix 3.

> **The Latitude of the Five Planets.**
> We require the total apogee motion and the apogee position, the mean position, the anomaly and the centrum. We find them all according to the method of the longitude of the five planets.

Commentary

The latitudes will be determined as a function of the mean anomaly and mean centrum, computed in the preceding sections according to Ptolemy's longitude model. Unlike most Greek and Islamic planetary latitude tables, in the *Huihui li* the latitude can be taken directly from a table with double arguments (see the following paper by van Dalen).

> **The definite anomaly.** We put down the signs, degrees and minutes of the anomaly. We multiply the signs by 10 and make degrees. For example, when we multiply 1 sign by 10 we obtain 10 degrees. This uses a simplifying method to reduce the calculation by which we construct the latitude table. We multiply the degrees by 20 and make minutes. In case sixties are filled we reduce them by division and make degrees. We also multiply the minutes by 20 and make seconds. In case sixties are filled we reduce them by division and make minutes. We add them, and thus we obtain it.

7 I have prepared an edition of the Arabic text of this book with an English translation and submitted it to Kyoto University as my D.Litt. dissertation. My study is to be published soon.

Commentary

The epicyclic (mean) anomaly (*zi xing gongdu fen* 自行宫度分), given by $\alpha = y_1^{3} y_2^{\circ} y'_3$, is scaled down to one third:

$$y = \frac{1}{3}\alpha = \frac{y_1^{s}}{3}\frac{y_2^{\circ}}{3}\frac{y'_3}{3} = 10y_1^{\circ}\, 20y'_2\, 20y''_3 .$$

Of course sexagesimal place value should be observed and therefore $20\,y'_2$ and $20\,y''_3$ should be reduced to suitable numbers if they exceed 60. The result (y) of this operation is called 'definite anomaly' (*zi xing dingdu* 自行定度). The terminology is confusing because the same term (自行定度) was already used (page 3762 line 11) to designate the 'true anomaly' and because no special marker was given to the 'degrees' etc. after this operation of scaling down.

The definite centrum. We put down the signs, degrees and minutes of the centrum. We multiply the signs by 5 and make degrees. For example, when we multiply 1 sign by 5 we obtain 5 degrees. We multiply the degrees by 10 and make minutes. In case sixties are filled we reduce them by division and make degrees. We also multiply the minutes by 10 and make seconds. In case sixties are filled we reduce them by division and make minutes. We add them, and thus we obtain it.

Commentary

The centrum (*xiaolun xin gongdu fen* 小輪心宫度分), given by $\gamma = x_1^{s} x_2^{\circ} x'_3$, is scaled down to one sixth:

$$x = \frac{1}{6}\gamma = \frac{x_1^{s}}{6}\frac{x_2^{\circ}}{6}\frac{x'_3}{6} = 5x_1^{\circ}\, 10x'_2\, 10x''_3 .$$

After taking care of sexagesimal place value as above, the result is called 'definite centrum' (*xiaolun xin dingdu* 小輪心定度). This is again misleading since the same term was already used (page 3762 line 11) to designate the 'true centrum.'

The latitude. With the definite centrum (x) and the definite anomaly (y) we enter the latitude table for the present planet and we take (a value, indicated by β_{mn} in Figure 1) (according to) both. One vertically, one horizontally.
We subtract the obtained number (β_{mn}) and (the number in) the following row ($\beta_{(m+1)n}$) from each other. If we happen to cross the ecliptic, we add (the obtained number) to (the number in) the following row.
Then we subtract the definite centrum (x) and the definite centrum at the top of the table (c_m) from each other. The horizontal row at the top. We multiply the remainders of the two subtractions and divide it by the increment (Δc) of the centrum at the top of the table. For instance, for Saturn each partition of the centrum in the horizontal row at the top is 3 degrees, for Mars each partition is 2 degrees.

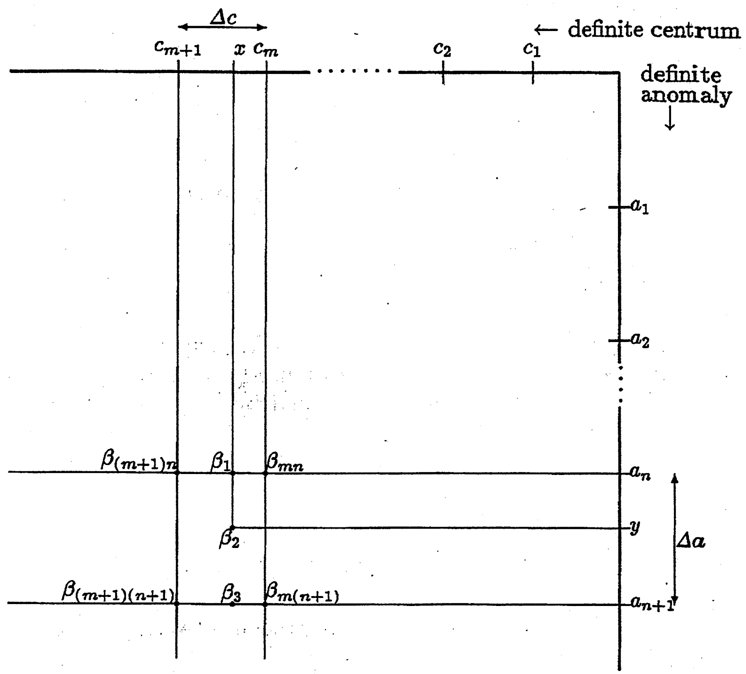

Figure 1. The layout of the two dimensional latitude table of the *Huihui li* and its interpolation scheme.

In case sixties are filled we take them together and make minutes. We add it to or subtract it from the number taken with both (arguments, i.e. β_{mn}). If it is more than (the number in) the following row we subtract, if less we add. In case we happen to cross the ecliptic, then if the number in the following row is more, we also subtract. We place it on the left.
Furthermore, we subtract the definite anomaly (y) and the definite anomaly at the top of the table (a_n) from each other. The first vertical row. Then we subtract the number taken with both (arguments, i.e. β_{mn}) and (the number in) the row below it ($\beta_{m(n+1)}$) from each other. If we happen to cross the ecliptic, we add it and the row below it. Then we multiply the remainders of the two subtractions. We divide it by the increment of the anomaly (Δa) at the top (i.e. the right) of the table. For instance, for Saturn each partition of the anomaly in the vertical row is 10 degrees, for Mars each partition is 4 degrees. We take it together and make minutes. These and the number previously placed on the left we add to or subtract from each other. If the number taken with both (arguments) is more than (the number in) the row below it we subtract, if less we add. If we happen to cross the ecliptic and the obtained minutes are more than the number placed on the left, we put down the obtained minutes and subtract the number placed on the left. The remainder is the minutes north or south that the ecliptic has been passed. Thus we obtain the definite ecliptical latitude.

Commentary

First we carry out a linear interpolation in the horizontal direction:

$$\beta_h = \frac{|\beta_{mn} - \beta_{(m+1)n}|(x - c_m)}{\Delta c}$$

$$\beta_1 = \beta_{mn} \pm \beta_h.$$

Then another interpolation has to be made in the vertical direction:

$$\beta_v = \frac{(y - a_n)|\beta_{mn} - \beta_{m(n+1)}|}{\Delta a}$$

$$\beta_2 \approx \beta_1 \pm \beta_v.$$

This is approximate because the correct vertical interpolation should be made between β_1 and β_3 (See Figure 1), instead of between β_{mn} and $\beta_{m(n+1)}$, and β_3 is to be computed by

$$\beta_3 = \beta_{m(n+1)} \pm \beta_h'$$

where

$$\beta_h' = \frac{|\beta_{m(n+1)} - \beta_{(m+1)(n+1)}|(x - c_m)}{\Delta c}$$

Now β_v' would be found by linear interpolation between β_1 and β_3 from:

$$\beta_v' = \frac{(y - a_n)|\beta_1 - \beta_3|}{\Delta a}$$

and finally we would get the correct value:

$$\beta_2 = \beta_1 \pm \beta_v'$$

From an inspection of the tabular values it can be seen that for Saturn, Jupiter and Mercury the maximum error which can be produced in the interpolation described in the *Huihui li* is smaller than 10′ and hence could hardly be observed. For Mars and Venus, however the error would reach half a degree in exceptional cases.

Detailed positions in latitude. We put down the latitude of the given planet in the section preceding (the desired latitude) and subtract it and the latitude at the following section from each other. We divide the remainder by the distance in days and make the day difference. We put down the latitude at the preceding section and we increase it by the day difference in the normal order or decrease it by it in the reverse order. The result is the latitude on successive days. We examine the latitude: if (the value at) the preceding section is less than (that at) the following section, we increase by the day difference in the normal order and decrease by it in the reverse order. If (the value at) the preceding section is more than (that at) the following section, we should decrease by the day difference in the normal order and increase by it in the reverse order. We cannot have one rule. In case (the values at) the preceding and following sections are not the same as far as north and south are concerned, we put down the latitude of the given planet at the preceding and the following section and add them. We divide it by the distance in days and make the day difference. We put down the latitude at the preceding section and repeatedly decrease it by the day difference until we cannot decrease it (any more). We subtract it from the day difference and repeatedly increase the remainder by the day difference. Thus we obtain the latitudes on successive days.

Commentary

The term 'detailed position' (*xi xing fen* 細行分) was already used in the section of the planetary longitude. In general, this is a simple, one-dimensional, linear interpolation. In this particular case, when the latitude β_n of a planet on one day and that on another day β_{n+d} after an interval of d days are given (called "sections" in our text), we first compute the 'day difference' (*richa* 日差) by

$$\Delta\beta = \frac{|\beta_n - \beta_{n+d}|}{d}$$

and, according to our main text, the result is successively added to or subtracted from β_n as the case may be, i.e., depending on whether $\beta_n \leq \beta_{n+d}$ or $\beta_n > \beta_{n+d}$. A special rule is also given for the cases when the directions of the two latitudes are different.

Appendix 1. Chinese Text[8]

五星緯度 求最高總行度, 中心行度, 自行度, 小輪心度, 並依五星經度術求之.

求自行定度 置自行宮度分, 其宮以十乘之爲度. 如一宮, 以二十乘之得十度, 此用約法折算, 以造緯度立成. 其度以二十乘之爲分, 滿六十約之爲度. 其分亦以二十乘之爲秒, 滿六十約之爲分. 併之卽得.

求小輪心定度 置小輪心宮度分, 其宮以五乘之爲度. 如一宮以五乘之, 得五度. 其度以一十乘之爲分, 滿六十約之爲度. 其分亦以一十乘之爲秒, 滿六十約之爲分. 併之卽得.

求緯度 以小輪心定度及自行定度, 入本星緯度立成內兩取, 一縱一横.

得數與後行相減. 若遇交黃道者, 與後行相併.

又以小輪心定度, 與立成上小輪心定度相減, 上横行.

兩減餘相乘, 以立成上小輪心度累加數除之. 如土星上横行小輪心度每隔三度, 火星每隔二度之類.

滿六十收之爲分, 用加減兩取數, 多於後行減, 少加. 若遇交黃道者, 卽後行數多亦減. 寄左.

復以自行定度與立成上自行定度相減, 首直行.

又以兩取數, 與下行相減, 若遇交黃道者, 與下行併. 兩減餘相乘, 以立成上自行度累加數除之, 如土星直行, 自行度每隔十度, 火星每隔四度之類.

收之爲分. 與前寄左數相加減, 如兩取數多於下行者減, 少加. 若遇交黃道者, 所得分多於寄左數, 置所得分內, 減寄左數, 餘爲交過黃道南北分也.

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8 *Lidai tianwen luli dengzhi huipian* 歷代天文律曆等志彙編 10, pp. 2764-65.

卽得黃道南北緯定分

求緯度細行分 置其星前段緯度, 與後段緯度相減, 餘以相距日除之, 爲日差.

置前段緯度, 以日差順加退減, 卽逐日緯度分. 按緯度前段少於後段者, 以日差順加退減. 若前段多於後段者, 宜以日差順減退加. 非可一例也.

若前後段南北不同者, 置其星前後段緯度併之, 以相距日除之, 爲日差.

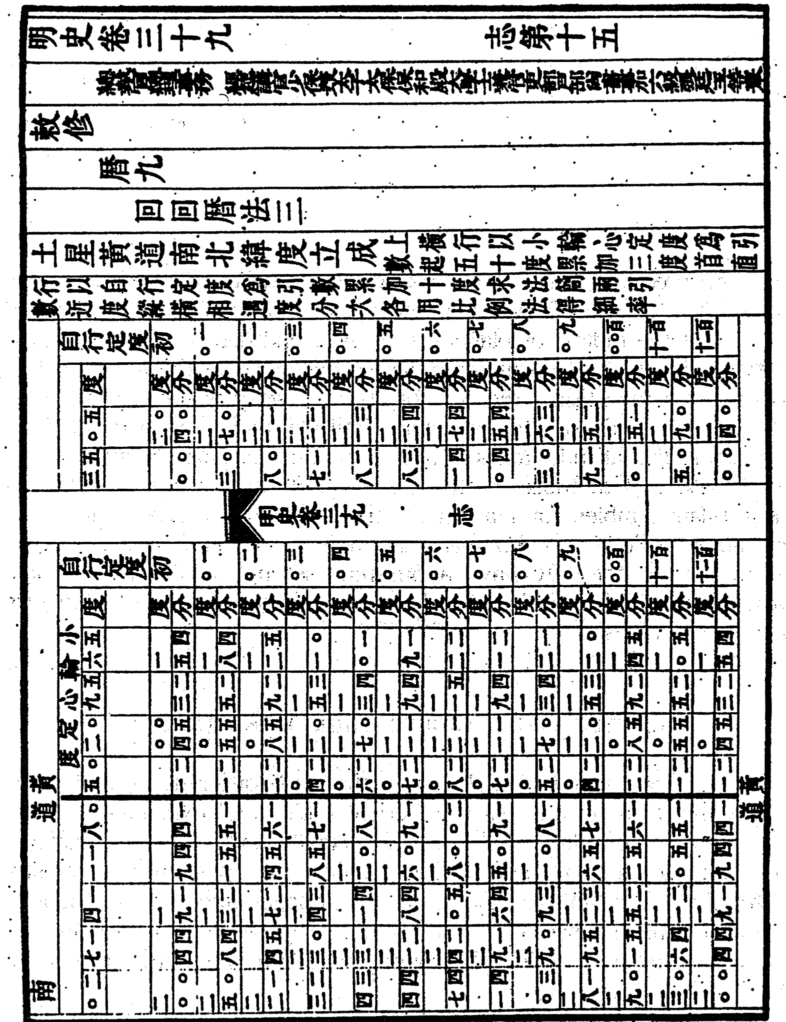
置前段緯度, 以日差累減之, 至不及減者, 於日差內減之, 餘以日差累加之, 卽得逐日緯度.

Appendix 2. Saturn's Latitude Table*

		definite anomaly (1/3)	0	10	20	30	40	50	60	70	80	90	100	110	120	
		deg.	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	
north		50	2 04	2 07	2 12	2 22[1]	2 32	2 42	2 47	2 45	2 36	2 25	2 15	2 09	2 04	north
		53	2 00	2 03	2 08	2 17	2 28	2 38	2 41	2 40	2 30[2]	2 19	2 10	2 05	2 00	
		56	1 45	1 48	1 52	2 01	2 10	2 19	2 22	2 21	2 12	2 02	1 54	1 50	1 45	
		59	1 23	1 25	1 29	1 35	1 43	1 49	1 51	1 49	1 43	1 35	1 29	1 25	1 23	
	definite centrum (1/6)	02	0 54	0 55	0 58	1 02	1 07	1 11	1 12	1 11	1 07	1 02	0 58	0 55	0 54	
north		05	0 21	0 21	0 22	0 24	0 26	0 27	0 28	0 27	0 25	0 24	0 22	0 21	0 21	north
ecl.		08	0 14	0 15	0 16	0 17	0 18	0 19	0 20	0 19	0 18	0 17	0 16	0 15	0 14	ecl.
south		11	0 49[3]	0 51[4]	0 54	0 58	1 02	1 06	1 08	1 05	1 01	0 56	0 52	0 50	0 49	south
		14	1 19	1 23	1 27	1 34	1 41	1 48	1 50	1 46	1 39	1 32[5]	1 25	1 21	1 19	
		17	1 44	1 48	1 54	2 03	2 13	2 21	2 24	2 19	2 09	1 59	1 51	1 46	1 44	
		20	2 00	2 05	2 12	2 23	2 34	2 44	2 47	2 41	2 30	2 18	2 09	2 03	2 00	
		23	2 04	2 09	2 14	2 24	2 35	2 45	2 47	2 43	2 31	2 22[6]	2 12	2 07	2 04	
		26	1 50	1 53	1 58	2 07	2 17	2 25	2 28	2 24	2 15	2 05	1 57	1 53	1 50	
		29	1 25	1 27	1 31	1 38	1 45	1 52	1 54	1 52	1 45	1 38	1 31	1 27	1 25	
		32	0 48	0 49	0 51	0 55	0 59	1 03	1 04	1 03	0 59	0 55	0 51	0 49	0 48	
south		35	0 06	0 06	0 07	0 07	0 07	0 08	0 08	0 08	0 08	0 07	0 07	0 06	0 06	south
ecl.		38	0 33	0 35	0 38	0 43	0 49	0 55	0 58	0 57	0 51	0 45	0 39	0 36	0 33	ecl.
north		41	1 09	1 11	1 14	1 19	1 26	1 32	1 36	1 34	1 28	1 22	1 16	1 12	1 09	north
		44	1 38[7]	1 40	1 45	1 52	2 02	2 10	2 15[8]	2 13	2 05	1 56	1 47	1 42	1 38	
		47	1 56	1 59	2 04	2 13	2 24	2 35	2 41	2 38	2 28	2 18	2 07	2 01	1 56	
		50	2 02	2 05	2 11	2 20	2 32	2 43	2 49	2 46	2 36	2 25	2 14	2 07	2 02	

* pp. 3819–22. The horizontal and vertical directions of the original table in the Chinese text (as seen in Fig.1 and Appendix 3) have been interchanged in this table. The abbreviations used in the following footnotes: C: 世宗實錄七政算 H: 明史回回曆 Q: 七政推步. [1] CQ 21; [2] H 03; [3] C 94; [4] H 15; [5] CQ 31; [6] C 21; [7] H 28; [8] C 25.

Appendix 3.



Tables of Planetary Latitude in the *Huihui li* (II)

Benno van DALEN*

In the preceding paper Professor Michio Yano contributed a general introduction to the *Huihui li* in which he explained the function and lay-out of the tables for planetary latitude, and demonstrated the type of two-dimensional linear interpolation described in the texts. In this paper I will investigate the possible Islamic sources for the *Huihui li* as a whole and for the planetary latitude tables in particular by comparing the latitude tables in a large number of Islamic astronomical hand-books with those in the *Huihui li*. It turns out that, whereas most Islamic tables for planetary latitude are minor modifications of the tables in Ptolemy's *Almagest*, the tables in the *Huihui li* are highly original: they not only have a different set-up, which makes them easier to use, but are also based on an entirely new set of planetary parameter values.

Islamic Astronomical Handbooks (*zījes*)

Between 800 and 1500 A.D. more than 200 different *zījes*, Islamic astronomical handbooks with numerical tables and explanatory texts, were compiled in places all over the Islamic world, from Afghanistan to Spain and from Yemen to China. These *zījes* were written mostly in Arabic or Persian, but also in other languages like Chinese, Hebrew or Latin. The thousands of manuscripts of *zījes* that are extant are now scattered in libraries all around the world. Very few have been published.[1]

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1. An extensive list of Islamic *zījes* together with a description of their contents can be found in Edward S. Kennedy, "A Survey of Islamic Astronomical Tables," *Transactions of the American Philosophical Society* 46(2) (1956), 123-77 (reprinted in 1989). Many important studies of tables from *zījes* are contained in Edward S. Kennedy *et al.*, *Studies in the Islamic Exact Sciences* (Beirut, 1983).

The earliest Islamic *zījes* relied strongly on Arabic translations of important astronomical works brought from India (in Sanskrit) or Byzantium (in Greek). From around 850 A.D. onwards most *zījes* were modeled after the *Almagest* or the *Handy Tables*, both written by Ptolemy, who not only summarized the most important achievements of his predecessors, but also was the first astronomer to design satisfactory geometrical models for the movement of the moon and the five planets visible to the naked eye. Early on, Muslim astronomers conducted extensive observational programs of their own in order to bring the parameter values (astronomical constants) underlying Ptolemy's tables up to date. They made various improvements to Ptolemy's planetary models and modified the set-up of his tables. Due to important advances in trigonometry, they were able to perform their calculations to a significantly higher degree of accuracy. In spite of their common methodological background, Islamic *zījes* exhibit many variations with respect to presentation, underlying parameter values, accuracy of tabular values, etc.

Unlike the preparation of Chinese calendars, the compilation of Islamic *zījes* was not a highly institutionalized, centralized activity. Various *zījes* were in use at the same time and place and usually there was not one particular *zīj* declared to be official. Many caliphs lacked interest in science in general or in astrology in particular and therefore they did not employ astronomers at all. *Zījes* were not normally used as plain calendars (i.e. for counting the days, months and years) as in China, but, for instance, played an important role in astrological predictions.

As a result, the development of Islamic astronomical tables was not as continuous as that of Chinese calendars. Periods of active astronomical study alternated with periods of hardly any significant achievements. Collaborative activities took place, in particular, under the caliph al-Maʾmūn in Baghdad and Damascus around 830 A.D., and in the large astronomical observatories of Maragha (northwestern Iran, c. 1260) and Samarkand (Uzbekistan, c. 1430). Some important *zījes*, like those of al-Battānī, Ibn Yūnus and al-Bīrūnī, were achievements of individuals working in isolation.

Another consequence of the fact that the compilation of Islamic *zījes* was not strongly centralized, is that in comparison with Chinese calendars their contents were standardized to a much lesser extent. However, most *zījes* contain at least tables and explanatory text for the following topics. Only in incidental cases are proofs for the presented methods and rules included.

- **Chronology**: descriptions of the various calendars in use in medieval Islam and the methods to convert dates from one calendar to another.

- **Trigonometrical functions**: sine, tangent and cotangent.
- **Spherical astronomy and timekeeping**: spherical coordinate conversions, determination of the time of the day from the positions of heavenly bodies, etc.
- **Planetary positions**: tables and instructions for the calculation of planetary longitudes and latitudes (see below). The planetary material in Islamic *zījes* was based on geometrical models and trigonometrical calculations, whereas Chinese calendars utilized various types of interpolation between observed values of the corrections to the mean planetary motions at certain intervals.
- **Lunar and solar eclipses.**
- **Astrology.**
- **Geography**: longitudes and latitudes of geographical localities.
- **Star table**: ecliptical longitudes and latitudes of fixed stars. Note that in Chinese astronomy such positions would be given with reference to the equator.

The Ptolemaic Planetary Models

As indicated above, most medieval Islamic astronomical handbooks relied on the planetary theory described by Ptolemy in his *Almagest*. Ptolemy presented geocentric geometrical models for the motion of the sun, moon and planets and explained in detail the trigonometrical calculations of the planetary positions according to these models. He compiled large sets of mathematical tables, which could be used to calculate many complicated astronomical phenomena at the cost of only a few additions and multiplications. These phenomena included, for instance, planetary positions, solar and lunar eclipses, and the visibility of the lunar crescent and the planets.[2]

(1) Longitude

Ptolemy constructed his planetary models on the basis of uniform circular motions, using the following elements:

① *eccentric motion* (Figure 1): the planet P moves uniformly on a circle whose center D is removed from the earth E. The distance DE is the *eccentricity*.

② *epicyclic motion* (Figure 2): the planet P moves uniformly on a small circle, called the *epicycle*. The centre C of the epicycle rotates around the earth E on a larger circle, called the *deferent* ("carrier").

[2] A Greek edition of the *Almagest* is available in J.L. Heiberg (ed.), *Claudii Ptolemaei, Opera quae exstant omnia, volumen 1. Syntaxis Mathematica* (Leipzig, 1898-1903). An English translation is contained in Gerald J. Toomer, *Ptolemy's Almagest* (London/New York, 1984). Explanations of the Ptolemaic planetary models can be found in O. Pedersen, *A Survey of the Almagest* (Odense, 1974); Otto E. Neugebauer, *A History of Ancient Mathematical Astronomy*, 3 vols. (Berlin, 1975).

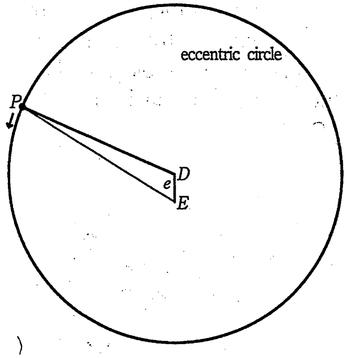

Figure 1. Eccentric model

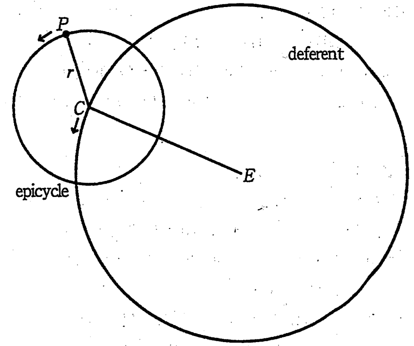

Figure 2. Epicycle model

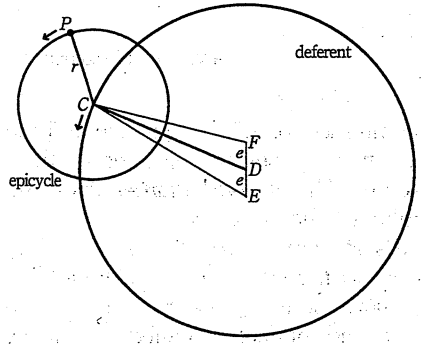

Figure 3. Planetary longitutes

In both cases the resulting motion of the planet as seen from the earth is non-uniform. In order to describe the complicated direct and retrograde motions of the five visible planets, Ptolemy needed a combination of an eccentric and an epicyclic motion (Figure 3). Thus, he let the planet P move uniformly on an epicycle with center C, which in its turn moved uniformly on a deferent whose center D was removed from the earth E. The agreement of the model with the actual planetary motion turned out to be better if the uniform motion of the epicycle center C took place not around the center D of the deferent, but around a point F twice as far removed from the earth as D in the same direction (in the case of Mercury the model is slightly different). As can be seen from Figure 3, the relative sizes of the planetary model are fixed as soon as the eccentricity DE and the radius of the epicycle CP are known in terms of the radius of the deferent DC.

The geometrical model thus having been established, the longitude of a planet could be calculated as a function of the position of the epicycle center on the deferent, the so-called *true centrum*, and the position of the planet on the epicycle, the *true anomaly*. These two quantities are found by adding a periodic correction, called an *equation*, to the *mean centrum* and *mean anomaly*, which are defined to be linear functions of time. Due to an ingenious type of interpolation, which has been named after Ptolemy, in the *Almagest* only 5 functions of a single variable had to be tabulated for each planet to enable the direct calculation of a planetary position from the two mean positions.

(2) Latitude

Whereas in a heliocentric planetary model a good approximation of the planetary motion in latitude can be obtained by simply tilting the planetary orbit slightly with respect to the orbit of the earth, the situation in Ptolemy's geocentric planetary models is much more complicated. In the *Almagest*, Ptolemy describes separate latitude models for the superior and the inferior planets. For both it will be convenient first to define the *first diameter* of the epicycle as the intersection of the epicycle and the plane perpendicular to the deferent which passes through the earth *E* and the epicycle center *C* (Figure 4). The *second diameter* of the epicycle is the diameter perpendicular to the first one.

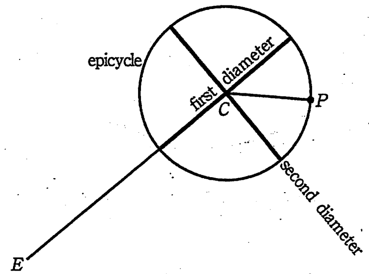

Figure 4. First and second diameters

In the case of the superior planets Ptolemy gives the plane of the deferent a small, constant inclination with respect to the ecliptic (i.e. the plane of the solar orbit). Furthermore, he gives the epicycle a small oscillatory movement around its second diameter, called *deviation*, which has a period equal to that of the motion of the epicycle center on the deferent. As a result, the latitude of a superior planet, moving through the half of its epicycle closest to the earth, will be larger than the latitude of that same planet when it moves through the half furthest from the earth. The underlying parameters of the latitude model for the superior planets are: the inclination of the plane of the deferent, the maximum deviation of the epicycle, and the position of the northernmost point of the deferent.

The latitude of the superior planets is a complicated function of both the true centrum and the true anomaly. Nevertheless, due to Ptolemaic interpolation, only three functions of a single variable need to be tabulated to enable a quick but accurate calculation of latitude values. Thus in the *Almagest* Ptolemy tabulates the latitude of the superior planets as a function of the true anomaly at the northernmost and at the southernmost point of the deferent (the so-called northern and southern *limits*), as well as an auxiliary function for performing Ptolemaic interpolation, to be called "interpolation function" in the remainder of this paper. Latitudes for intermediate positions of the epicycle center are obtained by simply multiplying one of the two limits by an appropriate value of the interpolation function. Table 1 displays part of the latitude tables in the *Jāmiʿ Zīj* by Kūshyār ibn Labbān (northern Iran, c. 1000), which are basically identical to those in the *Almagest*.

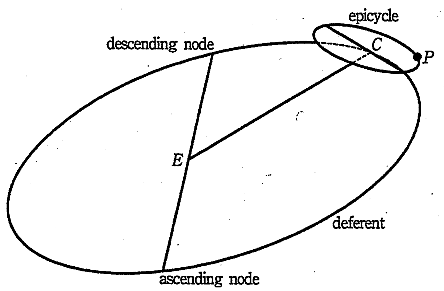

Figure 5. Planetary latitudes

Because the inferior planets have a more complicated motion in latitude, Ptolemy's model for these planets involves two more oscillations (Figure 5). Thus the inclination of the deferent is not any more a constant, but changes periodically. Furthermore, the epicycle oscillates both around its second diameter (*deviation*, similar to the superior planets) and around its first diameter (*slant*). The periods of the three oscillations are equal to that of the motion of the epicycle center on the deferent, the zero and maximum amplitudes occurring when the epicycle center is either in one of the two nodes (the intersecting points of the deferent and the ecliptic) or halfway between them. The underlying parameters of the latitude model for the inferior planets are: the maximum inclination of the deferent, the maximum deviation, and the maximum slant of the epicycle.

Unlike the superior planets, Ptolemy does not describe an exact method for the computation of the latitude of the inferior planets according to the above model. Instead, he gives instructions to calculate the three components describing the influence of the three oscillations on the latitude separately and then to add them. For this purpose he again needs to tabulate only three functions of a single variable: one for the deviation, one for the slant, and an interpolation function (cf. Table 1). The inclination of the deferent can simply be calculated using the interpolation function and explicitly given values of the maximum inclination.

Planetary Latitude Tables in Islamic Astronomy

We will now investigate the planetary latitude tables found in a large number of important Islamic *zījes*. Since most of the tables turn out to be directly related to those of Ptolemy, Table 2 below lists the characteristics of the tables with respect to the *Almagest*. The set-up

Table 1. Kūshyār ibn Labbān's Latitude Tables for Mars and Venus (from the manuscript Istanbul Fatih 3418, folios 82ʳ–82ᵛ).

Latitude of Mars					Latitude of Venus				
argument		northern limit	southern limit	interpol. function	argument		deviation	slant	interpol. function
6	354	0; 8	0; 4	60	6	354	1; 2	0; 8	60
12	348	0; 9	0; 4	59	12	348	1; 1	0;16	59
18	342	0;11	0; 5	57	18	342	1; 0	0;25	57
24	336	0;13	0; 6	55	24	336	0;59	0;33	55
30	330	0;14	0; 7	52	30	330	0;57	0;41	52
36	324	0;16	0; 9	48	36	324	0;55	0;49	48
42	318	0;18	0;12	44	42	318	0;51	0;57	44
48	312	0;21	0;15	40	48	312	0;46	1; 5	40
54	306	0;24	0;18	35	54	306	0;41	1;13	35
60	300	0;28	0;22	30	60	300	0;35	1;20	30
66	294	0;32	0;26	24	66	294	0;29	1;28	24
72	288	0;36	0;30	18	72	288	0;23	1;35	18
78	282	0;41	0;36	12	78	282	0;16	1;42	12
84	276	0;46	0;42	6	84	276	0; 8	1;50	6
90	270	0;52	0;49	0	90	270	0; 0	1;57	0
96	264	0;59	0;56	6	96	264	0;10	2; 3	6
102	258	1; 6	1; 4	12	102	258	0;20	2; 9	12
108	252	1;14	1;13	18	108	252	0;32	2;15	18
114	246	1;23	1;24	24	114	246	0;45	2;20	24
120	240	1;34	1;37	30	120	240	1; 0	2;24	30
126	234	1;46	1;53	35	126	234	1;19	2;27	35
132	228	2; 0	2;13	40	132	228	1;40	2;29	40
138	222	2;16	2;35	44	138	222	2; 5	2;30	44
144	216	2;34	3; 1	48	144	216	2;32	2;28	48
150	210	2;54	3;29	52	150	210	3; 3	2;22	52
156	204	3;15	4; 3	55	156	204	3;37	2;12	55
162	198	3;35	4;41	57	162	198	4;14	1;55	57
168	192	3;52	5;25	59	168	192	4;53	1;27	59
174	186	4; 8	6;13	60	174	186	5;36	0;48	60
180	180	4;21	7; 7	60	180	180	6;20	0; 0	60

The planetary latitude tables in the *Jāmiʿ Zīj* by Kūshyār (northern Persia, c. 1000 A.D.) are almost identical copies of the tables in the *Almagest*. However, some parts of the tables for Mars and Venus, shown above, show various differences, which seem to be due either to scribal errors or to a partial recomputation of the table by Kūshyār. The arguments are given in degrees and denote the true anomaly of the planet in the case of the northern and southern limits, the deviation and the slant. In the case of the interpolation functions the argument is the true centrum plus or minus a constant.

of the tables is represented by letters from A to M in the *Modifications* column and by the letter A or a number in the *Arguments* column. The meaning of all letters is explained below. In general, the letters A to C indicate latitude tables of the base types found in the *Almagest*, *Handy Tables* and Indian sources respectively. The letters D to M

Table 2. Important Islamic astronomical handbooks (*zījes*) and the characteristics of their tables for planetary latitude.

Zīj	*Astronomer*	*Locality*	*Date (A.D.)*	*Latitude tables*	
				Modifications	*Arguments*
Almagest	Ptolemy	Alexandria (Egypt)	140	A	A
Handy Tables	Ptolemy	Alexandria	150	B	3
Mumtaḥan Zīj	Yaḥyā b. Abī Manṣūr	Baghdad (Iraq)	830	C	1
Sindhind Zīj	al-Khwārizmī	Baghdad	840	C	1
Damascene Zīj	Ḥabash al-Ḥāsib	Damascus (Syria)	840	A	A
Ṣābiʾ Zīj	al-Battānī	Raqqa (Syria)	900	A	6
Jāmiʿ Zīj	Kūshyār b. Labbān	northern Iran	970	A	6
Ḥākimī Zīj	Ibn Yūnus	Cairo (Egypt)	1000	A	1
Masudic Canon	al-Bīrūnī	Ghazna (Afgh.)	1030	AH	1
Toledan Tables	al-Zarqālī	Toledo (Spain)	1075	C	1
Sanjarī Zīj	al-Khāzinī	Mary (Turkm.)	1120	ADEFG	1
Muqtabis Zīj	Ibn al-Kammād	Cordoba (Spain)	1130	AB	6
Tunisian Zīj	Ibn Isḥāq	Morocco	1220	A / L	1 / 12
Shāmil Zīj		northwestern Iran	1240	ADEFG	1
Īlkhānī Zīj	al-Ṭūsī	Maragha (Iran)	1260	ADEFG	6
Huihui li		Beijing (China)	1275	KM	various
	al-Maghribī	Maragha	1280	AHIJK	6
Ashrafī Zīj	al-Kamālī	central Iran	1310	ADEFG	1
Jadīd Zīj	Ibn al-Shāṭir	Damascus	1350	A	A
	al-Sanjufīnī	Tibet	1366	ADGK	6
Khāqānī Zīj	al-Kāshī	Samarkand (Uzb.)	1420	ADE / M	A / 5
Sulṭānī Zīj	Ulugh Beg	Samarkand	1440	ADEHJK	A

A=*Almagest*, **B**=*Handy Tables*, **C**=Indian methods, **D**=shift of interpolation function, **E**=first latitude, **F**=two slant tables, **G**=interpolation function to minutes, **H**=newly calculated interpolation function, **I**=one function for the northern and southern limits of Saturn and Jupiter, **J**=new values of the nodes, **K**=new values of the other latitude parameters, **L**=double-argument table as a function of the true centrum and true anomaly, **M**=double-argument table as a function of the mean centrum and mean anomaly.

indicate Islamic modifications of the *Almagest* tables.

(1) Base Types

A. *The standard type of the Almagest.* As has been described above, the *Almagest* tabulates the northern and southern limits for each of the superior planets, the inclination and slant for Venus and Mercury, and an interpolation function common to all planets. All eleven functions involved are tabulated for arguments 6, 12, 18, ..., 90, 93, 96, ..., 180°; the interpolation function is nothing more than a sine.[3]

B. *The type found in the Handy Tables.* Whereas the *Almagest* was

3 Heiberg, *op. cit.*, pp. 582-86; Toomer, *op. cit.*, pp. 632-34.

of a more theoretical nature, another set of astronomical tables by Ptolemy, the *Handy Tables*, was more practical and easier to use. The planetary latitudes were among the very few instances in which Ptolemy decided to give the *Handy Tables* a structure completely different from the *Almagest* and to base them on different values of the underlying parameters.[4] Muslim astronomers almost exclusively adopted the latitude tables from the *Almagest* rather than those from the *Handy Tables*.

AB. *A mixture of the Almagest tables for the superior and the Handy Tables for the inferior planets.* This occurs in the *Muqtabis Zīj* by Ibn al-Kammād (c. 1130) with only one small modification of the underlying parameter values for the inferior planets.[5]

C. *Indian types.* These can be found in some *zījes* written in the first half of the 9th century. For instance, the latitude tables in the famous *Mumtaḥan Zīj* by Yaḥyā ibn Abī Manṣūr (c. 830) display simple sine functions with linear interpolation between independently calculated values for multiples of 15°. A more complicated latitude model of Indian origin was used in the *zīj* of al-Khwārizmī (c. 840). In this work two latitude functions were tabulated for each planet, one of which had to be divided by the other.[6]

(2) Modifications of the *Almagest* Tables by Muslim Astronomers

D. *Shifted interpolation function.* Before the values of the interpolation function for the superior planets can be taken from the latitude tables in the *Almagest*, a constant must be added to or subtracted from their argument. In various Islamic works this addition or subtraction was avoided by shifting the values of the interpolation function in the table by this constant amount with respect to the argument.

E. *Tabulation of the "first latitude."* As was explained above, the latitude of the inferior planets is found as the sum of three components, which, in Arabic and Persian sources, were called the first, second, and third latitudes. In the set-up of the *Almagest*, the first latitude (inclination) cannot be found directly from the tables, but has to be calculated separately. In various Islamic astronomical handbooks, however, we find explicit tabulations of the first latitude.

F. *Two tables for the slant of Mercury.* In calculating the latitude of Mercury, the slant as given in the *Almagest* (with maximum value 2;30) has to be increased or decreased by a tenth depending on the position of the epicycle center on the deferent. In some Islamic *zījes*, these cases have been separated and two tables for the slant are presented with maximum values 2;15 and 2;45 respectively.

G. *Rounding of the interpolation function.* In the *Almagest* the values of the interpolation function are given in minutes and seconds. In some medieval sources the *Almagest* values were rounded to minutes.

H. *New calculation of the interpolation function.* The values of the interpolation function in the *Almagest* are supposed to be ordinary sine values, but were not accurately calculated. Some medieval astronomers recalculated the interpolation function with greater accuracy.

I. *A single function for the northern and southern limit.* al-Maghribī, active at the observatory founded by Hulagu Khan in Maragha, combined the northern and southern limits of Saturn and Jupiter into one table, probably because he realized that they are practically equal.

J. *New values for the position of the nodes.* The latitude tables in some *zījes* involved different values for the position of the nodes of the superior planets. These required slight changes in the set-up of the tables or in the method of using them, but had no influence on the tabular values themselves.

K. *New values of inclination, deviation and slant.* For the *Huihui li*, the *zīj* of al-Sanjufīnī (Tibet, 1366), and the *Sulṭānī Zīj* by Ulugh Beg (Samarkand, c. 1440) calculations of planetary latitude tables were carried out on the basis of entirely new sets of parameter values. In a couple of other *zījes* only some of the parameter values were modified.

(3) Double-argument Tables

Since the planetary latitude is a function of two variables—the centrum (position of the epicycle center on the deferent) and the anomaly (position of the planet on the epicycle)—it can be found directly from a table having these two quantities as a *double argument*. In case the arguments of such a table are the *true* centrum and *true* anomaly, these must first be calculated from the mean centrum and mean anomaly before the table can be used. If the arguments are the *mean* positions, the table has a more complicated structure and is therefore more difficult to compute. On the other hand, the determination of latitudes from the table requires even less effort.

L. *Double-argument tables with the true centrum and true anomaly*

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[4] William D. Stahlman, "The Astronomical Tables of Codex Vaticanus Graecus 1291" (Doctoral thesis, Brown University, 1959), pp. 143-55 and 325-34.

[5] José Chabás and Bernard R. Goldstein, "Andalusian Astronomy: al-Zīj al-Muqtabis of Ibn al-Kammād," *Archive for History of Exact Sciences* 48 (1994), 1-41, esp. pp. 31-32.

[6] Mercè Viladrich, "The Planetary Latitude Tables in the Mumtaḥan Zīj," *Journal for the History of Astronomy* 19 (1988), 257-68; Otto Neugebauer, *The Astronomical Tables of al-Khwārizmī* (Copenhagen, 1962), pp. 34-41; Edward S. Kennedy and Walid Ukashah, "al-Khwārizmī's Planetary Latitude Tables," *Centaurus* 14 (1969), 86-96.

as arguments. This type of tables can be found in the *Tunisian Zīj* by Ibn Isḥāq, who worked in Morocco in the early 13th century. As we will see below, Ibn Isḥāq's tables were computed directly from the latitude tables in the *Almagest*, which are also contained in the *Tunisian Zīj*.

M. *Double-argument tables with the mean centrum and mean anomaly as arguments*. As we have seen, tables of this type are contained in the *Huihui li*. In his *Khāqānī Zīj* the computational genius al-Kāshī (Samarkand, c. 1420) not only presented the *Almagest* tables, but besides intended to calculate a full set of tables with the mean centrum and mean anomaly as a double argument (see below).

(4) Modifications to the Range of Arguments in the *Almagest*

A. *Almagest range*. As was mentioned above, the latitude tables in the *Almagest* were drawn up for arguments 6, 12, 18, ..., 90, 93, 96, ..., 180°, a range which is typical for this work.

6. *Multiples of 6°*. Some Muslim astronomers left out the *Almagest* values for odd multiples of 3° and tabulated the latitude only for each multiple of 6°.

3. *Multiples of 3°*. In the *Handy Tables* Ptolemy tabulated the planetary latitude functions for each 3 degrees of the arguments.

1. *Every single degree*. Many Muslim astronomers tabulated the planetary latitudes for each single degree of the arguments. In some cases it can be verified that they used (linear) interpolation between the values from the *Almagest*.

(5) Developments

From Table 2 we can conclude that Ptolemy's tables for planetary latitude in the *Almagest* basically survived the Islamic Middle Ages in their original form. It is not surprising that early Muslim astronomers like Ḥabash al-Ḥāsib, al-Battānī and Kūshyār simply copied the *Almagest* tables, but Ibn Yūnus, al-Bīrūnī and later authors of *zījes* certainly had the capability to calculate planetary latitude tables of their own. The modifications of the *Almagest* tables which were made by al-Khāzinī (c. 1120) and various successors of his (letters **D** till **J**) were elementary and cannot be considered important innovations.

There are various possible explanations for the survival of the planetary latitude tables from the *Almagest* throughout the Middle Ages.

① Ptolemy's latitude theory and the calculations required in order to draw up latitude tables according to his theory were so complicated that only highly skilled astronomers would be inclined to modify the theory and/or tables. This is, for instance, illustrated by Kūshyār's highly confused and incorrect demonstration of the calculation of planetary latitudes in Book IV of his *Jāmiʿ Zīj*. Even late in the Middle Ages, Muslim astronomers depended on a translation of the *Almagest* itself for a reliable description of Ptolemy's latitude theory.

② Ptolemy's latitude tables might have been accurate enough for the purposes of medieval astronomers. In this connection it can be noted that the accuracy of medieval observational instruments was not much higher than 10 minutes of arc. As far as we know, no investigation has ever been made of the significance of planetary latitudes in Islamic astronomy or the accuracy of medieval calculations and observations of planetary latitudes.

③ The influence of small changes in the planetary parameters (like the eccentricity and the epicycle radius) can hardly be noticed in calculated latitude values. Therefore a new computation of latitude tables might have been considered unnecessary even when the planetary parameters were changed.

The only Islamic tables for planetary latitude which do contain significant innovations with respect to Ptolemy's tables can be recognized in Table 2 by one or more of the letters **K**, **L** and **M** in the *Modifications* column. Of these, the tables in the *zīj* of al-Sanjufīnī (see below) and in the *Sulṭānī Zīj* by Ulugh Beg (Samarkand, c. 1440) were of the same structure as the *Almagest* tables, but were largely based on new parameter values (modification **K**). The three sets of latitude tables with a completely different structure (**L**, **M**) will now be discussed separately.

Ibn Isḥāq. The *Tunisian Zīj* by Ibn Isḥāq (Morocco, c. 1220) contains the earliest double-argument tables for planetary latitude that we have found in our investigation. The arguments of these tables are the true centrum and true anomaly of the planets, which had to be calculated from the corresponding mean positions before the table could be used. It can be verified that Ibn Isḥāq calculated the double-argument tables directly from the latitude tables in the *Almagest*, which are also contained in the *Tunisian Zīj*. This was a relatively easy task, since no planetary equations had to be considered and the resulting tables are partially symmetric (unlike tables that have the mean planetary positions as arguments).

Besides double-argument tables for planetary latitudes, the *Tunisian Zīj* also has the value 23°32′30″ for the obliquity of the ecliptic in common with the *Huihui li*. Furthermore, it has been suggested that one of the earliest Islamic astronomical works written in Chinese, the *Madaba li* by Yelü Chucai (a high official of Genghis Khan in Samarkand around 1220),[7] might have been named after the *Muqtabis*

[7] Cf. Kiyosi Yabuuti, "The Influence of Islamic Astronomy in China," in D.A. King and G.A. Saliba (eds.) *From Deferent to Equant* (New York: The New

Zīj by the Andalusian astronomer Ibn al-Kammād (12th century), who is known to have influenced Ibn Isḥāq. Therefore it is tempting to suppose that Islamic astronomy in China derived from western-Islamic sources. However, inspection of the survey of the *Muqtabis Zīj* by Chabás and Goldstein (see note 5) rules out this possibility at once: the *Muqtabis Zīj* is partially based on Ptolemaic material and partially on typically western-Islamic planetary models and parameter values, no traces of which can be found in the *Huihui li*.

al-Kāshī. Similar to Ibn Isḥāq, al-Kāshī (Samarkand, c. 1420) presented both the planetary latitude tables from the *Almagest* and a set of double-argument tables in his *Khāqānī Zīj*. Unlike Ibn Isḥāq, al-Kāshī's tables had the mean centrum and mean anomaly of the planets as arguments, thus even further simplifying the calculation of the latitudes. However, the task of computing these tables might have been too formidable for al-Kāshī himself, since in two extant manuscripts of his *zīj* the tables have not been finished.[8]

Conclusion

From the above overview of tables for planetary latitude in Islamic *zījes* we can conclude that the *Huihui li* contains the earliest double-argument tables with the mean planetary motions as arguments, thus making the calculation of latitudes as easy as possible, as well as the earliest systematic modifications of the underlying planetary parameter values. Together with other novelties in the *Huihui li*, like changes in the set-up of the planetary equations and a star table based on completely new observations, this shows that the Muslim astronomers at the Imperial Astronomical Bureaus in Beijing and Nanjing were both very capable and highly innovative.

The Relationship between the *Huihui li* and the *Sanjufīnī Zīj*

There is one Arabic astronomical work which is clearly related to the *Huihui li*, namely the *zīj* written by a certain Abū Muḥammad ʿAṭā ibn Aḥmad ibn Muḥammad Khwāja Ghāzī al-Sanjufīnī in 1366. Although the author originated from the region around Samarkand, he worked for the Mongol viceroy of Tibet, a direct descendant in the seventh generation from Genghis Khan. The manuscript contains notes in Chinese, apparently made by a librarian, marginal glosses in Mongolian, and explanatory text with about 50 astronomical tables in Arabic.[9]

The *Sanjufīnī Zīj* has various tables in common with the *Huihui li*, namely those for the oblique ascension (the rising time of a given arc of the ecliptic at a specific geographical latitude), the equation of time, the lunar latitude, the lunar and planetary equations, the planetary stations, and a table for parallax. Furthermore, the *Sanjufīnī Zīj* contains a table for planetary latitudes of the *Almagest* type, but based on the same non-Ptolemaic parameter values as the latitude tables in the *Huihui li*. Because it seems implausible that one would reconstruct a table of *Almagest* type from a double-argument table, we may assume that the tables for planetary latitude in the *Huihui li* were calculated from those in the *Sanjufīnī Zīj*. It seems possible that the tables in the *Sanjufīnī Zīj* stemmed from an earlier version of the *Huihui li* than is now extant. In that case, the double-argument table in the *Huihui li* would be a Ming modification rather than an original Yuan achievement. A further investigation of the *Sanjufīnī Zīj* will be necessary in order to determine the precise relationship between the two works.

York Academy of Sciences, 1987), pp. 547-59, esp. pp. 547-48. See also Kiyoshi Yabuuti (translated and partially revised by Benno van Dalen), "Islamic Astronomy in China during the Yuan and Ming Dynasties," *Historia Scientiarum* 7 (1977), 11-43.

8 Edward S. Kennedy, *On the Contents and Significance of the Khāqānī Zīj by Jamshid al-Dīn al-Kāshī* (Frankfurt am Mein, 1998), pp. 29-30.

9 The *Sanjufīnī Zīj* is extant in the unique manuscript Paris Bibliothèque Nationale arabe 6040. Philological aspects of the manuscript were studied in Herbert Franke, "Mittelmongolische Glossen in einer arabischen astronomischen Handschrift," *Oriens* 31 (1988), 95-118. Some of the tables in the *Sanjufīnī Zīj* were investigated in the following articles: Edward S. Kennedy, "Eclipse Predictions in Arabic Astronomical Tables Prepared for the Mongol Viceroy of Tibet," *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften* 4 (1987/88), 60-80; Edward S. Kennedy and Jan P. Hogendijk, "Two Tables from an Arabic Astronomical Handbook for the Mongol Viceroy of Tibet," in E. Leichty, M. de J. Ellis *et al.* (eds.) *A Scientific Humanist* (Philadelphia, 1988), pp. 233-42; Benno van Dalen, "A Statistical Method for Recovering Unknown Parameters from Medieval Astronomical Tables," *Centaurus* 32 (1989), 98-106.