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Tables of Planetary Latitude in the *Huihui li* (II)

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In the preceding paper Professor Michio Yano contributed a general introduction to the *Huihui li* in which he explained the function and lay-out of the tables for planetary latitude, and demonstrated the type of two-dimensional linear interpolation described in the texts. In this paper I will investigate the possible Islamic sources for the *Huihui li* as a whole and for the planetary latitude tables in particular by comparing the latitude tables in a large number of Islamic astronomical hand-books with those in the *Huihui li*. It turns out that, whereas most Islamic tables for planetary latitude are minor modifications of the tables in Ptolemy's *Almagest*, the tables in the *Huihui li* are highly original: they not only have a different set-up, which makes them easier to use, but are also based on an entirely new set of planetary parameter values.

Islamic Astronomical Handbooks (*zījes*)

Between 800 and 1500 A.D. more than 200 different *zījes*, Islamic astronomical handbooks with numerical tables and explanatory texts, were compiled in places all over the Islamic world, from Afghanistan to Spain and from Yemen to China. These *zījes* were written mostly in Arabic or Persian, but also in other languages like Chinese, Hebrew or Latin. The thousands of manuscripts of *zījes* that are extant are now scattered in libraries all around the world. Very few have been published.[1]

The earliest Islamic *zījes* relied strongly on Arabic translations of important astronomical works brought from India (in Sanskrit) or Byzantium (in Greek). From around 850 A.D. onwards most *zījes* were modeled after the *Almagest* or the *Handy Tables*, both written by Ptolemy, who not only summarized the most important achievements of his predecessors, but also was the first astronomer to design satisfactory geometrical models for the movement of the moon and the five planets visible to the naked eye. Early on, Muslim astronomers conducted extensive observational programs of their own in order to bring the parameter values (astronomical constants) underlying Ptolemy's tables up to date. They made various improvements to Ptolemy's planetary models and modified the set-up of his tables. Due to important advances in trigonometry, they were able to perform their calculations to a significantly higher degree of accuracy. In spite of their common methodological background, Islamic *zījes* exhibit many variations with respect to presentation, underlying parameter values, accuracy of tabular values, etc.

Unlike the preparation of Chinese calendars, the compilation of Islamic *zījes* was not a highly institutionalized, centralized activity. Various *zījes* were in use at the same time and place and usually there was not one particular *zīj* declared to be official. Many caliphs lacked interest in science in general or in astrology in particular and therefore they did not employ astronomers at all. *Zījes* were not normally used as plain calendars (i.e. for counting the days, months and years) as in China, but, for instance, played an important role in astrological predictions.

As a result, the development of Islamic astronomical tables was not as continuous as that of Chinese calendars. Periods of active astronomical study alternated with periods of hardly any significant achievements. Collaborative activities took place, in particular, under the caliph al-Maʾmūn in Baghdad and Damascus around 830 A.D., and in the large astronomical observatories of Maragha (northwestern Iran, c. 1260) and Samarkand (Uzbekistan, c. 1430). Some important *zījes*, like those of al-Battānī, Ibn Yūnus and al-Bīrūnī, were achievements of individuals working in isolation.

Another consequence of the fact that the compilation of Islamic *zījes* was not strongly centralized, is that in comparison with Chinese calendars their contents were standardized to a much lesser extent. However, most *zījes* contain at least tables and explanatory text for the following topics. Only in incidental cases are proofs for the presented methods and rules included.

- **Chronology**: descriptions of the various calendars in use in medieval Islam and the methods to convert dates from one calendar to another.

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[1] An extensive list of Islamic *zījes* together with a description of their contents can be found in Edward S. Kennedy, "A Survey of Islamic Astronomical Tables," *Transactions of the American Philosophical Society* 46(2) (1956), 123-77 (reprinted in 1989). Many important studies of tables from *zījes* are contained in Edward S. Kennedy *et al.*, *Studies in the Islamic Exact Sciences* (Beirut, 1983).

- **Trigonometrical functions**: sine, tangent and cotangent.
- **Spherical astronomy and timekeeping**: spherical coordinate conversions, determination of the time of the day from the positions of heavenly bodies, etc.
- **Planetary positions**: tables and instructions for the calculation of planetary longitudes and latitudes (see below). The planetary material in Islamic *zījes* was based on geometrical models and trigonometrical calculations, whereas Chinese calendars utilized various types of interpolation between observed values of the corrections to the mean planetary motions at certain intervals.
- **Lunar and solar eclipses.**
- **Astrology.**
- **Geography**: longitudes and latitudes of geographical localities.
- **Star table**: ecliptical longitudes and latitudes of fixed stars. Note that in Chinese astronomy such positions would be given with reference to the equator.

The Ptolemaic Planetary Models

As indicated above, most medieval Islamic astronomical handbooks relied on the planetary theory described by Ptolemy in his *Almagest*. Ptolemy presented geocentric geometrical models for the motion of the sun, moon and planets and explained in detail the trigonometrical calculations of the planetary positions according to these models. He compiled large sets of mathematical tables, which could be used to calculate many complicated astronomical phenomena at the cost of only a few additions and multiplications. These phenomena included, for instance, planetary positions, solar and lunar eclipses, and the visibility of the lunar crescent and the planets.[2]

(1) Longitude

Ptolemy constructed his planetary models on the basis of uniform circular motions, using the following elements:

① *eccentric motion* (Figure 1): the planet P moves uniformly on a circle whose center D is removed from the earth E. The distance DE is the *eccentricity*.

② *epicyclic motion* (Figure 2): the planet P moves uniformly on a small circle, called the *epicycle*. The centre C of the epicycle rotates around the earth E on a larger circle, called the *deferent* ("carrier").

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[2] A Greek edition of the *Almagest* is available in J.L. Heiberg (ed.), *Claudii Ptolemaei, Opera quae exstant omnia, volumen 1. Syntaxis Mathematica* (Leipzig, 1898-1903). An English translation is contained in Gerald J. Toomer, *Ptolemy's Almagest* (London/New York, 1984). Explanations of the Ptolemaic planetary models can be found in O. Pedersen, *A Survey of the Almagest* (Odense, 1974); Otto E. Neugebauer, *A History of Ancient Mathematical Astronomy*, 3 vols. (Berlin, 1975).

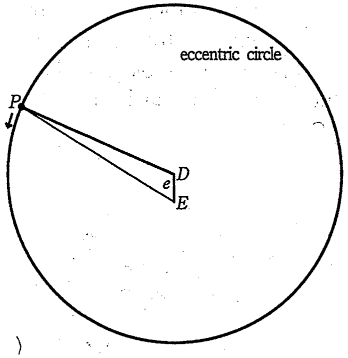

Figure 1. Eccentric model

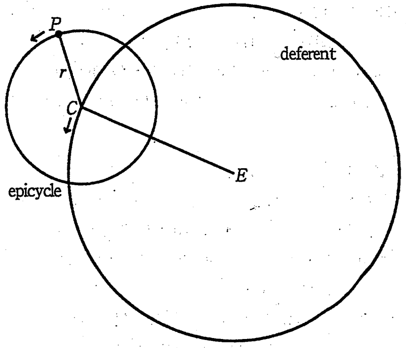

Figure 2. Epicycle model

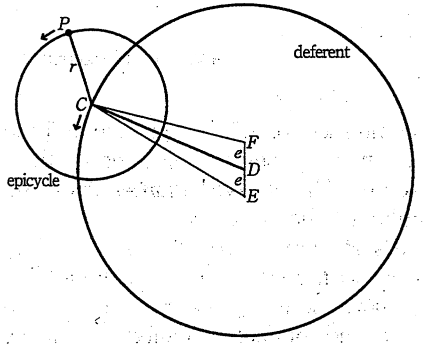

Figure 3. Planetary longitutes

In both cases the resulting motion of the planet as seen from the earth is non-uniform.

In order to describe the complicated direct and retrograde motions of the five visible planets, Ptolemy needed a combination of an eccentric and an epicyclic motion (Figure 3). Thus, he let the planet P move uniformly on an epicycle with center C, which in its turn moved uniformly on a deferent whose center D was removed from the earth E. The agreement of the model with the actual planetary motion turned out to be better if the uniform motion of the epicycle center C took place not around the center D of the deferent, but around a point F twice as far removed from the earth as D in the same direction (in the case of Mercury the model is slightly different). As can be seen from Figure 3, the relative sizes of the planetary model are fixed as soon as the eccentricity DE and the radius of the epicycle CP are known in terms of the radius of the deferent DC.

The geometrical model thus having been established, the longitude of a planet could be calculated as a function of the position of the epicycle center on the deferent, the so-called *true centrum*, and the position of the planet on the epicycle, the *true anomaly*. These two quantities are found by adding a periodic correction, called an *equation*, to the *mean centrum* and *mean anomaly*, which are defined to be linear functions of time. Due to an ingenious type of interpolation, which has been named after Ptolemy, in the *Almagest* only 5 functions of a single variable had to be tabulated for each planet to enable the direct calculation of a planetary position from the two mean positions.

(2) Latitude

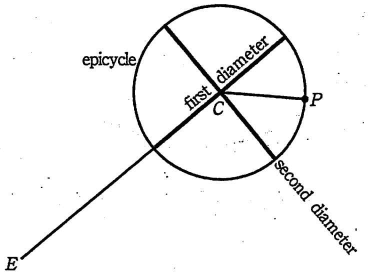

Figure 4. First and second diameters

Whereas in a heliocentric planetary model a good approximation of the planetary motion in latitude can be obtained by simply tilting the planetary orbit slightly with respect to the orbit of the earth, the situation in Ptolemy's geocentric planetary models is much more complicated. In the *Almagest*, Ptolemy describes separate latitude models for the superior and the inferior planets. For both it will be convenient first to define the *first diameter* of the epicycle as the intersection of the epicycle and the plane perpendicular to the deferent which passes through the earth E and the epicycle center C (Figure 4). The *second diameter* of the epicycle is the diameter perpendicular to the first one.

In the case of the superior planets Ptolemy gives the plane of the deferent a small, constant inclination with respect to the ecliptic (i.e. the plane of the solar orbit). Furthermore, he gives the epicycle a small oscillatory movement around its second diameter, called *deviation*, which has a period equal to that of the motion of the epicycle center on the deferent. As a result, the latitude of a superior planet, moving through the half of its epicycle closest to the earth, will be larger than the latitude of that same planet when it moves through the half furthest from the earth. The underlying parameters of the latitude model for the superior planets are: the inclination of the plane of the deferent, the maximum deviation of the epicycle, and the position of the northernmost point of the deferent.

The latitude of the superior planets is a complicated function of both the true centrum and the true anomaly. Nevertheless, due to Ptolemaic interpolation, only three functions of a single variable need to be tabulated to enable a quick but accurate calculation of latitude values. Thus in the *Almagest* Ptolemy tabulates the latitude of the superior planets as a function of the true anomaly at the northernmost and at the southernmost point of the deferent (the so-called northern and southern *limits*), as well as an auxiliary function for performing Ptolemaic interpolation, to be called "interpolation function" in the remainder of this paper. Latitudes for intermediate positions of the epicycle center are obtained by simply multiplying one of the two limits by an appropriate value of the interpolation function. Table 1 displays part of the latitude tables in the *Jāmiʿ Zīj* by Kūshyār ibn Labbān (northern Iran, c. 1000), which are basically identical to those in the *Almagest*.

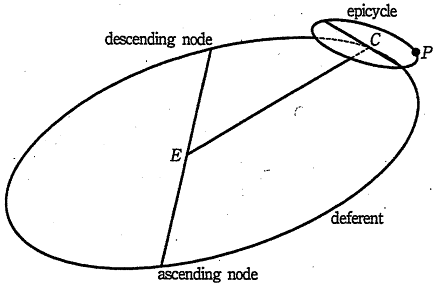

Figure 5. Planetary latitudes

Because the inferior planets have a more complicated motion in latitude, Ptolemy's model for these planets involves two more oscillations (Figure 5). Thus the inclination of the deferent is not any more a constant, but changes periodically. Furthermore, the epicycle oscillates both around its second diameter (*deviation*, similar to the superior planets) and around its first diameter (*slant*). The periods of the three oscillations are equal to that of the motion of the epicycle center on the deferent, the zero and maximum amplitudes occurring when the epicycle center is either in one of the two nodes (the intersecting points of the deferent and the ecliptic) or halfway between them. The underlying parameters of the latitude model for the inferior planets are: the maximum inclination of the deferent, the maximum deviation, and the maximum slant of the epicycle.

Unlike the superior planets, Ptolemy does not describe an exact method for the computation of the latitude of the inferior planets according to the above model. Instead, he gives instructions to calculate the three components describing the influence of the three oscillations on the latitude separately and then to add them. For this purpose he again needs to tabulate only three functions of a single variable: one for the deviation, one for the slant, and an interpolation function (cf. Table 1). The inclination of the deferent can simply be calculated using the interpolation function and explicitly given values of the maximum inclination.

Planetary Latitude Tables in Islamic Astronomy

We will now investigate the planetary latitude tables found in a large number of important Islamic *zījes*. Since most of the tables turn out to be directly related to those of Ptolemy, Table 2 below lists the characteristics of the tables with respect to the *Almagest*. The set-up

Table 1. Kūshyār ibn Labbān's Latitude Tables for Mars and Venus (from the manuscript Istanbul Fatih 3418, folios 82ʳ–82ᵛ).

Latitude of Mars					Latitude of Venus				
argument		northern limit	southern limit	interpol. function	argument		deviation	slant	interpol. function
6	354	0; 8	0; 4	60	6	354	1; 2	0; 8	60
12	348	0; 9	0; 4	59	12	348	1; 1	0;16	59
18	342	0;11	0; 5	57	18	342	1; 0	0;25	57
24	336	0;13	0; 6	55	24	336	0;59	0;33	55
30	330	0;14	0; 7	52	30	330	0;57	0;41	52
36	324	0;16	0; 9	48	36	324	0;55	0;49	48
42	318	0;18	0;12	44	42	318	0;51	0;57	44
48	312	0;21	0;15	40	48	312	0;46	1; 5	40
54	306	0;24	0;18	35	54	306	0;41	1;13	35
60	300	0;28	0;22	30	60	300	0;35	1;20	30
66	294	0;32	0;26	24	66	294	0;29	1;28	24
72	288	0;36	0;30	18	72	288	0;23	1;35	18
78	282	0;41	0;36	12	78	282	0;16	1;42	12
84	276	0;46	0;42	6	84	276	0; 8	1;50	6
90	270	0;52	0;49	0	90	270	0; 0	1;57	0
96	264	0;59	0;56	6	96	264	0;10	2; 3	6
102	258	1; 6	1; 4	12	102	258	0;20	2; 9	12
108	252	1;14	1;13	18	108	252	0;32	2;15	18
114	246	1;23	1;24	24	114	246	0;45	2;20	24
120	240	1;34	1;37	30	120	240	1; 0	2;24	30
126	234	1;46	1;53	35	126	234	1;19	2;27	35
132	228	2; 0	2;13	40	132	228	1;40	2;29	40
138	222	2;16	2;35	44	138	222	2; 5	2;30	44
144	216	2;34	3; 1	48	144	216	2;32	2;28	48
150	210	2;54	3;29	52	150	210	3; 3	2;22	52
156	204	3;15	4; 3	55	156	204	3;37	2;12	55
162	198	3;35	4;41	57	162	198	4;14	1;55	57
168	192	3;52	5;25	59	168	192	4;53	1;27	59
174	186	4; 8	6;13	60	174	186	5;36	0;48	60
180	180	4;21	7; 7	60	180	180	6;20	0; 0	60

The planetary latitude tables in the *Jāmiʿ Zīj* by Kūshyār (northern Persia, c. 1000 A.D.) are almost identical copies of the tables in the *Almagest*. However, some parts of the tables for Mars and Venus, shown above, show various differences, which seem to be due either to scribal errors or to a partial recomputation of the table by Kūshyār. The arguments are given in degrees and denote the true anomaly of the planet in the case of the northern and southern limits, the deviation and the slant. In the case of the interpolation functions the argument is the true centrum plus or minus a constant.

Table 2. Important Islamic astronomical handbooks (*zījes*) and the characteristics of their tables for planetary latitude.

Zīj	*Astronomer*	*Locality*	*Date (A.D.)*	*Latitude tables*	
				Modifications	*Arguments*
Almagest	Ptolemy	Alexandria (Egypt)	140	A	A
Handy Tables	Ptolemy	Alexandria	150	B	3
Mumtaḥan Zīj	Yaḥyā b. Abī Manṣūr	Baghdad (Iraq)	830	C	1
Sindhind Zīj	al-Khwārizmī	Baghdad	840	C	1
Damascene Zīj	Ḥabash al-Ḥāsib	Damascus (Syria)	840	A	A
Ṣābiʾ Zīj	al-Battānī	Raqqa (Syria)	900	A	6
Jāmiʿ Zīj	Kūshyār b. Labbān	northern Iran	970	A	6
Ḥākimī Zīj	Ibn Yūnus	Cairo (Egypt)	1000	A	1
Masudic Canon	al-Bīrūnī	Ghazna (Afgh.)	1030	AH	1
Toledan Tables	al-Zarqālī	Toledo (Spain)	1075	C	1
Sanjarī Zīj	al-Khāzinī	Mary (Turkm.)	1120	ADEFG	1
Muqtabis Zīj	Ibn al-Kammād	Cordoba (Spain)	1130	AB	6
Tunisian Zīj	Ibn Isḥāq	Morocco	1220	A / L	1 / 12
Shāmil Zīj		northwestern Iran	1240	ADEFG	1
Īlkhānī Zīj	al-Ṭūsī	Maragha (Iran)	1260	ADEFG	6
Huihui li		Beijing (China)	1275	KM	various
	al-Maghribī	Maragha	1280	AHIJK	6
Ashrafī Zīj	al-Kamālī	central Iran	1310	ADEFG	1
Jadīd Zīj	Ibn al-Shāṭir	Damascus	1350	A	A
	al-Sanjufīnī	Tibet	1366	ADGK	6
Khāqānī Zīj	al-Kāshī	Samarkand (Uzb.)	1420	ADE / M	A / 5
Sulṭānī Zīj	Ulugh Beg	Samarkand	1440	ADEHJK	A

A=*Almagest*, **B**=*Handy Tables*, **C**=Indian methods, **D**=shift of interpolation function, **E**=first latitude, **F**=two slant tables, **G**=interpolation function to minutes, **H**=newly calculated interpolation function, **I**=one function for the northern and southern limits of Saturn and Jupiter, **J**=new values of the nodes, **K**=new values of the other latitude parameters, **L**=double-argument table as a function of the true centrum and true anomaly, **M**=double-argument table as a function of the mean centrum and mean anomaly.

of the tables is represented by letters from A to M in the *Modifications* column and by the letter A or a number in the *Arguments* column. The meaning of all letters is explained below. In general, the letters A to C indicate latitude tables of the base types found in the *Almagest*, *Handy Tables* and Indian sources respectively. The letters D to M indicate Islamic modifications of the *Almagest* tables.

(1) Base Types

A. *The standard type of the Almagest.* As has been described above, the *Almagest* tabulates the northern and southern limits for each of the superior planets, the inclination and slant for Venus and Mercury, and an interpolation function common to all planets. All eleven functions involved are tabulated for arguments 6, 12, 18, ..., 90, 93, 96, ..., 180°; the interpolation function is nothing more than a sine.[3]

B. *The type found in the Handy Tables.* Whereas the *Almagest* was

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[3] Heiberg, *op. cit.*, pp. 582–86; Toomer, *op. cit.*, pp. 632–34.

of a more theoretical nature, another set of astronomical tables by Ptolemy, the *Handy Tables*, was more practical and easier to use. The planetary latitudes were among the very few instances in which Ptolemy decided to give the *Handy Tables* a structure completely different from the *Almagest* and to base them on different values of the underlying parameters.[4] Muslim astronomers almost exclusively adopted the latitude tables from the *Almagest* rather than those from the *Handy Tables*.

AB. *A mixture of the Almagest tables for the superior and the Handy Tables for the inferior planets.* This occurs in the *Muqtabis Zīj* by Ibn al-Kammād (c. 1130) with only one small modification of the underlying parameter values for the inferior planets.[5]

C. *Indian types.* These can be found in some *zījes* written in the first half of the 9th century. For instance, the latitude tables in the famous *Mumtaḥan Zīj* by Yaḥyā ibn Abī Manṣūr (c. 830) display simple sine functions with linear interpolation between independently calculated values for multiples of 15°. A more complicated latitude model of Indian origin was used in the *zīj* of al-Khwārizmī (c. 840). In this work two latitude functions were tabulated for each planet, one of which had to be divided by the other.[6]

(2) Modifications of the *Almagest* Tables by Muslim Astronomers

D. *Shifted interpolation function.* Before the values of the interpolation function for the superior planets can be taken from the latitude tables in the *Almagest*, a constant must be added to or subtracted from their argument. In various Islamic works this addition or subtraction was avoided by shifting the values of the interpolation function in the table by this constant amount with respect to the argument.

E. *Tabulation of the "first latitude."* As was explained above, the latitude of the inferior planets is found as the sum of three components, which, in Arabic and Persian sources, were called the first, second, and third latitudes. In the set-up of the *Almagest*, the first latitude (inclination) cannot be found directly from the tables, but has to be calculated separately. In various Islamic astronomical handbooks, however, we find explicit tabulations of the first latitude.

F. *Two tables for the slant of Mercury.* In calculating the latitude of Mercury, the slant as given in the *Almagest* (with maximum value 2;30) has to be increased or decreased by a tenth depending on the position of the epicycle center on the deferent. In some Islamic *zījes*, these cases have been separated and two tables for the slant are presented with maximum values 2;15 and 2;45 respectively.

G. *Rounding of the interpolation function.* In the *Almagest* the values of the interpolation function are given in minutes and seconds. In some medieval sources the *Almagest* values were rounded to minutes.

H. *New calculation of the interpolation function.* The values of the interpolation function in the *Almagest* are supposed to be ordinary sine values, but were not accurately calculated. Some medieval astronomers recalculated the interpolation function with greater accuracy.

I. *A single function for the northern and southern limit.* al-Maghribī, active at the observatory founded by Hulagu Khan in Maragha, combined the northern and southern limits of Saturn and Jupiter into one table, probably because he realized that they are practically equal.

J. *New values for the position of the nodes.* The latitude tables in some *zījes* involved different values for the position of the nodes of the superior planets. These required slight changes in the set-up of the tables or in the method of using them, but had no influence on the tabular values themselves.

K. *New values of inclination, deviation and slant.* For the *Huihui li*, the *zīj* of al-Sanjufīnī (Tibet, 1366), and the *Sulṭānī Zīj* by Ulugh Beg (Samarkand, c. 1440) calculations of planetary latitude tables were carried out on the basis of entirely new sets of parameter values. In a couple of other *zījes* only some of the parameter values were modified.

(3) Double-argument Tables

Since the planetary latitude is a function of two variables—the centrum (position of the epicycle center on the deferent) and the anomaly (position of the planet on the epicycle)—it can be found directly from a table having these two quantities as a *double argument*. In case the arguments of such a table are the *true* centrum and *true* anomaly, these must first be calculated from the mean centrum and mean anomaly before the table can be used. If the arguments are the *mean* positions, the table has a more complicated structure and is therefore more difficult to compute. On the other hand, the determination of latitudes from the table requires even less effort.

L. *Double-argument tables with the true centrum and true anomaly*

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[4] William D. Stahlman, "The Astronomical Tables of Codex Vaticanus Graecus 1291" (Doctoral thesis, Brown University, 1959), pp. 143-55 and 325-34.

[5] José Chabás and Bernard R. Goldstein, "Andalusian Astronomy: al-Zīj al-Muqtabis of Ibn al-Kammād," *Archive for History of Exact Sciences* 48 (1994), 1-41, esp. pp. 31-32.

[6] Mercè Viladrich, "The Planetary Latitude Tables in the Mumtaḥan Zīj," *Journal for the History of Astronomy* 19 (1988), 257-68; Otto Neugebauer, *The Astronomical Tables of al-Khwārizmī* (Copenhagen, 1962), pp. 34-41; Edward S. Kennedy and Walid Ukashah, "al-Khwārizmī's Planetary Latitude Tables," *Centaurus* 14 (1969), 86-96.

as arguments. This type of tables can be found in the *Tunisian Zīj* by Ibn Isḥāq, who worked in Morocco in the early 13th century. As we will see below, Ibn Isḥāq's tables were computed directly from the latitude tables in the *Almagest*, which are also contained in the *Tunisian Zīj*.

M. *Double-argument tables with the mean centrum and mean anomaly as arguments.* As we have seen, tables of this type are contained in the *Huihui li*. In his *Khāqānī Zīj* the computational genius al-Kāshī (Samarkand, c. 1420) not only presented the *Almagest* tables, but besides intended to calculate a full set of tables with the mean centrum and mean anomaly as a double argument (see below).

(4) Modifications to the Range of Arguments in the *Almagest*

A. *Almagest range.* As was mentioned above, the latitude tables in the *Almagest* were drawn up for arguments 6, 12, 18, ..., 90, 93, 96, ..., 180°, a range which is typical for this work.

6. *Multiples of 6°.* Some Muslim astronomers left out the *Almagest* values for odd multiples of 3° and tabulated the latitude only for each multiple of 6°.

3. *Multiples of 3°.* In the *Handy Tables* Ptolemy tabulated the planetary latitude functions for each 3 degrees of the arguments.

1. *Every single degree.* Many Muslim astronomers tabulated the planetary latitudes for each single degree of the arguments. In some cases it can be verified that they used (linear) interpolation between the values from the *Almagest*.

(5) Developments

From Table 2 we can conclude that Ptolemy's tables for planetary latitude in the *Almagest* basically survived the Islamic Middle Ages in their original form. It is not surprising that early Muslim astronomers like Ḥabash al-Ḥāsib, al-Battānī and Kūshyār simply copied the *Almagest* tables, but Ibn Yūnus, al-Bīrūnī and later authors of *zījes* certainly had the capability to calculate planetary latitude tables of their own. The modifications of the *Almagest* tables which were made by al-Khāzinī (c. 1120) and various successors of his (letters **D** till **J**) were elementary and cannot be considered important innovations.

There are various possible explanations for the survival of the planetary latitude tables from the *Almagest* throughout the Middle Ages.

① Ptolemy's latitude theory and the calculations required in order to draw up latitude tables according to his theory were so complicated that only highly skilled astronomers would be inclined to modify the theory and/or tables. This is, for instance, illustrated by Kūshyār's highly confused and incorrect demonstration of the calculation of planetary latitudes in Book IV of his *Jāmiʿ Zīj*. Even late in the Middle Ages, Muslim astronomers depended on a translation of the *Almagest* itself for a reliable description of Ptolemy's latitude theory.

② Ptolemy's latitude tables might have been accurate enough for the purposes of medieval astronomers. In this connection it can be noted that the accuracy of medieval observational instruments was not much higher than 10 minutes of arc. As far as we know, no investigation has ever been made of the significance of planetary latitudes in Islamic astronomy or the accuracy of medieval calculations and observations of planetary latitudes.

③ The influence of small changes in the planetary parameters (like the eccentricity and the epicycle radius) can hardly be noticed in calculated latitude values. Therefore a new computation of latitude tables might have been considered unnecessary even when the planetary parameters were changed.

The only Islamic tables for planetary latitude which do contain significant innovations with respect to Ptolemy's tables can be recognized in Table 2 by one or more of the letters **K**, **L** and **M** in the *Modifications* column. Of these, the tables in the *zīj* of al-Sanjufīnī (see below) and in the *Sulṭānī Zīj* by Ulugh Beg (Samarkand, c. 1440) were of the same structure as the *Almagest* tables, but were largely based on new parameter values (modification **K**). The three sets of latitude tables with a completely different structure (**L**, **M**) will now be discussed separately.

Ibn Isḥāq. The *Tunisian Zīj* by Ibn Isḥāq (Morocco, c. 1220) contains the earliest double-argument tables for planetary latitude that we have found in our investigation. The arguments of these tables are the true centrum and true anomaly of the planets, which had to be calculated from the corresponding mean positions before the table could be used. It can be verified that Ibn Isḥāq calculated the double-argument tables directly from the latitude tables in the *Almagest*, which are also contained in the *Tunisian Zīj*. This was a relatively easy task, since no planetary equations had to be considered and the resulting tables are partially symmetric (unlike tables that have the mean planetary positions as arguments).

Besides double-argument tables for planetary latitudes, the *Tunisian Zīj* also has the value 23°32′30″ for the obliquity of the ecliptic in common with the *Huihui li*. Furthermore, it has been suggested that one of the earliest Islamic astronomical works written in Chinese, the *Madaba li* by Yelü Chucai (a high official of Genghis Khan in Samarkand around 1220),[7] might have been named after the *Muqtabis*

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[7] Cf. Kiyosi Yabuuti, "The Influence of Islamic Astronomy in China," in D.A. King and G.A. Saliba (eds.) *From Deferent to Equant* (New York: The New

Zīj by the Andalusian astronomer Ibn al-Kammād (12th century), who is known to have influenced Ibn Isḥāq. Therefore it is tempting to suppose that Islamic astronomy in China derived from western-Islamic sources. However, inspection of the survey of the *Muqtabis Zīj* by Chabás and Goldstein (see note 5) rules out this possibility at once: the *Muqtabis Zīj* is partially based on Ptolemaic material and partially on typically western-Islamic planetary models and parameter values, no traces of which can be found in the *Huihui li*.

al-Kāshī. Similar to Ibn Isḥāq, al-Kāshī (Samarkand, c. 1420) presented both the planetary latitude tables from the *Almagest* and a set of double-argument tables in his *Khāqānī Zīj*. Unlike Ibn Isḥāq, al-Kāshī's tables had the mean centrum and mean anomaly of the planets as arguments, thus even further simplifying the calculation of the latitudes. However, the task of computing these tables might have been too formidable for al-Kāshī himself, since in two extant manuscripts of his *zīj* the tables have not been finished.[8]

Conclusion

From the above overview of tables for planetary latitude in Islamic *zījes* we can conclude that the *Huihui li* contains the earliest double-argument tables with the mean planetary motions as arguments, thus making the calculation of latitudes as easy as possible, as well as the earliest systematic modifications of the underlying planetary parameter values. Together with other novelties in the *Huihui li*, like changes in the set-up of the planetary equations and a star table based on completely new observations, this shows that the Muslim astronomers at the Imperial Astronomical Bureaus in Beijing and Nanjing were both very capable and highly innovative.

The Relationship between the *Huihui li* and the *Sanjufīnī Zīj*

There is one Arabic astronomical work which is clearly related to the *Huihui li*, namely the *zīj* written by a certain Abū Muḥammad ʿAṭā ibn Aḥmad ibn Muḥammad Khwāja Ghāzī al-Sanjufīnī in 1366. Although the author originated from the region around Samarkand, he worked for the Mongol viceroy of Tibet, a direct descendant in the seventh generation from Genghis Khan. The manuscript contains notes in Chinese, apparently made by a librarian, marginal glosses in Mongolian, and explanatory text with about 50 astronomical tables in Arabic.[9]

The *Sanjufīnī Zīj* has various tables in common with the *Huihui li*, namely those for the oblique ascension (the rising time of a given arc of the ecliptic at a specific geographical latitude), the equation of time, the lunar latitude, the lunar and planetary equations, the planetary stations, and a table for parallax. Furthermore, the *Sanjufīnī Zīj* contains a table for planetary latitudes of the *Almagest* type, but based on the same non-Ptolemaic parameter values as the latitude tables in the *Huihui li*. Because it seems implausible that one would reconstruct a table of *Almagest* type from a double-argument table, we may assume that the tables for planetary latitude in the *Huihui li* were calculated from those in the *Sanjufīnī Zīj*. It seems possible that the tables in the *Sanjufīnī Zīj* stemmed from an earlier version of the *Huihui li* than is now extant. In that case, the double-argument table in the *Huihui li* would be a Ming modification rather than an original Yuan achievement. A further investigation of the *Sanjufīnī Zīj* will be necessary in order to determine the precise relationship between the two works.

York Academy of Sciences, 1987), pp. 547-59, esp. pp. 547-48. See also Kiyoshi Yabuuti (translated and partially revised by Benno van Dalen), "Islamic Astronomy in China during the Yuan and Ming Dynasties," *Historia Scientiarum* 7 (1977), 11-43.

[8] Edward S. Kennedy, *On the Contents and Significance of the Khāqānī Zīj by Jamshid al-Dīn al-Kāshī* (Frankfurt am Mein, 1998), pp. 29-30.

[9] The *Sanjufīnī Zīj* is extant in the unique manuscript Paris Bibliothèque Nationale arabe 6040. Philological aspects of the manuscript were studied in Herbert Franke, "Mittelmongolische Glossen in einer arabischen astronomischen Handschrift," *Oriens* 31 (1988), 95-118. Some of the tables in the *Sanjufīnī Zīj* were investigated in the following articles: Edward S. Kennedy, "Eclipse Predictions in Arabic Astronomical Tables Prepared for the Mongol Viceroy of Tibet," *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften* 4 (1987/88), 60-80; Edward S. Kennedy and Jan P. Hogendijk, "Two Tables from an Arabic Astronomical Handbook for the Mongol Viceroy of Tibet," in E. Leichty, M. de J. Ellis *et al.* (eds.) *A Scientific Humanist* (Philadelphia, 1988), pp. 233-42; Benno van Dalen, "A Statistical Method for Recovering Unknown Parameters from Medieval Astronomical Tables," *Centaurus* 32 (1989), 98-106.