FROM CHINA TO PARIS: 2000 YEARS TRANSMISSION OF MATHEMATICAL IDEAS

EDITED BY

YVONNE DOLD-SAMPLONIUS
JOSEPH W. DAUBEN
MENSO FOLKERTS
BENNO VAN DALEN

FRANZ STEINER VERLAG STUTTGART
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Islamic and Chinese Astronomy under the Mongols: a Little-Known Case of Transmission

by BENNO VAN DALEN

Thanks to the Mongol conquests in the 13th century, a scientific exchange between the Iranian part of the Islamic world on the one hand and the Yuan Dynasty in China on the other became possible. This exchange resulted in the use of a Chinese type of lunisolar calendar in Iran and the construction of instruments and compilation of handbooks with tables by Muslim astronomers in China. In this article, we describe the exchange of astronomical knowledge between Muslims and Chinese in the Mongol period in some detail. In order to obtain more insight into the process of transmission, we first sketch the historical background of the exchange. In an Appendix, methods for investigating the relationships between astronomical tables are described.

1. Introduction: Investigating Transmission

This volume contains many examples in which the transmission of mathematical theorems, algorithms, problems, and their solutions is plausible, but cannot be established with certainty. This is a difficulty often encountered in the study of transmission of early mathematics. On the one hand, we find hardly any explicit historical information on the origin of the contents of mathematical works. On the other hand, a comparison of mathematical works from different cultures usually does not allow us to decide with certainty whether similar theorems, problems and solutions were transmitted, or independently invented or constructed.

The present article deals with a little-known case of transmission of mathematical ideas, namely that of mathematical-astronomical knowledge between the Islamic world and China in the Mongol period (13th century). In judging to what extent Chinese astronomers took over Islamic astronomical methods in their own calendars and instruments, and vice versa, we encounter the problem sketched above: we find hardly any explicit attributions, and a direct comparison of astronomical handbooks and instruments does not lead to unambiguous conclusions concerning possible connections.

Four specific cases of scientific contact between Muslim and Chinese astronomers in the 13th century are discussed below. In these cases, attention will be paid to three aspects in particular that may provide information important for judging the possibility of transmission:

1. The historical background of the transmission and the types of contacts that were possible between Muslims and Chinese;

2. Detailed technical descriptions of the objects to be compared, namely astronomical handbooks with tables and various types of astronomical instruments, and of the way in which they were used;
3. Mathematical methods for testing the similarity of astronomical tables and the possibility of connections between them.

The historical information (see Section 2 below) makes clear that there were ample opportunities for contact between Muslims and Chinese. In fact, Muslims in Mongol China were so common that it need not surprise us that the scientific level at the Islamic Astronomical Bureau in Beijing was very high. A detailed technical description of the properties of the so-called "Chinese-Uighur calendar" (Section 3.2) allows us to make well-founded statements about its origin. Finally, the mathematical methods described in the Appendix for testing the relationships between astronomical tables turn out to be exceptionally useful tools. Since the tabular values are highly idiosyncratic, it is often possible to decide whether two tables for the same function stem from the same source (in spite of incidental differences), and, with the aid of statistical or numerical methods, whether a table for a given function was calculated from another table for a different function.

2. Historical Background: A Short History of the Mongol Expansion in the 13th Century

In the first decade of the 13th century, Chinggis Khan united the Mongol tribes living on the steppes to the north of the Great Wall and started the expansion of what would soon become the Mongol world empire. His first raids against China resulted in the conquest of the capital of the Jin dynasty, present-day Beijing, in 1215. Chinggis then turned his attention to the west, occupied the province of Khwarezm (south of the Aral Sea) and, in 1220, took the important cities of Bukhara and Samarkand (in present-day Uzbekistan) without resistance. However, neither in Northern China, nor in Transoxiana was the Mongol rule solidly established at this stage.

After Chinggis's death in 1227, the first division of the Mongol empire took place between his four sons. The third son, Ögödei, became Chinggis's successor as Great Khan. Together with the youngest son Tolui, he achieved the final submission of the Jin dynasty in 1234. The oldest son, Jochi, received the lands north of the Caspian Sea, which stayed under Mongol reign for several centuries under the name Golden Horde. The spectacular campaign against Russia and Eastern Europe, which only came to a halt due to the death of Ögödei in 1241, was led by Jochi's son Batu. At that time, the Mongol empire stretched from Hungary in the west to Korea in the east.

From 1251 onwards the most important roles in the Mongol empire were played by the sons of Tolui. His oldest son Möngke was Great Khan from 1251 to 1259 and extended the capital Karakorum in Central Mongolia with a number of palaces designed by Chinese, Iranian, and Russian architects. Möngke's brother and successor Khubilai Khan, Tolui's second son, continued the conquest of China. He made Beijing the new capital of the Mongol empire and named himself the first emperor of the Yuan dynasty. In 1279 he finally defeated the Southern Song dynasty and reunified China under one rule.

Meanwhile, in the 1250s, the third son of Tolui, Hülegü, had led extensive military campaigns in the Middle East. By taking Baghdad in 1258 he made an end to the Abbasid dynasty and as the first Ilkhan ("Submissive" Khan) he reigned over Iran and Iraq from 1256 to 1265. After the death of Khubilai in 1294 and the final conversion of the Ilkhans to Islam in the following year, the Ilkhans became practically independent from the Mongol empire. Their power gradually decreased by the middle of the 14th century, when they were replaced by the Timurids. In China the Mongolian Yuan dynasty was defeated and replaced by the Ming in 1368.[1]

2.1 Foreigners in the Service of the Mongols. The Mongols were good warriors, but no rulers. Therefore they entrusted the administration of their empire to loyal subjects of the peoples they subjugated. Already in the first decade of the thirteenth century the Uighurs, a highly civilized Turkic people, became subordinates of the Mongols. The Uighurs had lived in present-day Xinjiang (China's westernmost province) for more than 300 years and had converted to Islam in the tenth century. In their submission to Chinggis Khan they saw the possibility of escaping the suppression exerted on them by the Western Liao dynasty. They soon became to play an important role in the Mongol administration and, since the Uighur alphabet was adopted for the Mongol language, also in education.

In 1218, after his first campaign in China, Chinggis Khan made Yelü Chucai (1189–1243), a Chinese statesman of Mongol descent, his personal advisor. Yelü was the most influential Chinese in the Mongol administration and accompanied Chinggis on all of his expeditions. When the Mongols planned the definite conquest of northern China in the early 1230s, it was Yelü who convinced Ögödei Khan that it would be more profitable to raise taxes from the Chinese population than to lay waste the whole country and reduce it to pastures for the horses.

In particular during the campaigns against the Southern Song dynasty in the 1260s and 1270s, tens of thousands of Muslims arrived in China. Many came voluntarily to pursue a career in the Mongol administration where they served, for instance, as tax collectors. Craftsmen, artists and scholars were forced to settle in China, where they left traces in such areas as pottery, poetry, music, architecture, medicine, astronomy, and military technology. Muslim merchants had frequented the Chinese trade centres on the Silk Road and on the coast before the Mongol conquest, but now benefited particularly from the fast and safe connections within the Mongol empire.

Among the Muslims in China were Uighurs and other people from territories bordering on China, but also, thanks to the good relations between the Yuan dynasty and the Ilkhanate, very large numbers of Iranians. As a result, Persian became one of the main languages of the Mongol administration in China. The Muslims mostly lived in separate quarters of the towns, where they were allowed to build mosques

1 More information on the Mongols and their conquests in the 13th century can be found, e.g., in [Spuler 1972]. The Persian history of the Mongols written in the late 13th century by the famous historian Rashīd al-Dīn was partially translated into English in [Boyle 1971].

and to practice their religion with only very few limitations. Tibetans also played an important role under Khubilai Khan and exerted a strong spiritual, Lamaistic influence on the Mongols.[2]

2.2 Exchange of Astronomical Knowledge under the Mongols. Within the historical context sketched above, the following contacts between Muslim and Chinese astronomers are known to have taken place:

- During the "western expedition" of Chinggis Khan (ca. 1220), Yelü Chucai learned of Islamic astronomical handbooks with tables, so-called *zīj*es, and used some techniques from them to adjust the official Chinese calendar.[3]
- The Great Khan Möngke (1251–1259) had plans to build an observatory in the Mongol capital Karakorum. For this purpose he intended to make use of the service of a Muslim astronomer from Bukhara named Jamāl al-Dīn Muḥammad ibn Ṭāhir ibn Muḥammad al-Zaydī. It is very probable that this is the same person referred to as Zhamaluding in Chinese annals from the year 1267 onwards. Due to various problems Möngke's plans were never realized.[4]
- At the observatory of Maragha, founded in 1258 by Hülegü, a Chinese astronomer is known to have been active. It is plausible that he was largely responsible for the descriptions of the so-called Chinese-Uighur calendar found in many Persian *zīj*es from the Mongol period onwards.
- In 1271 Khubilai Khan founded in his capital Beijing an Islamic Astronomical Bureau with an observatory that operated parallel to the official Chinese Astronomical Bureau, and whose first director was the above-mentioned Zhamaluding. Based on newly made observations, the Muslims active at the Bureau compiled a *zīj*, which is extant in a Chinese translation of 1383.

2 The role of the Muslims in Yuan China is discussed in various contributions to [Langlois 1981], in particular in M. Rossabi, *The Muslims in the Early Yüan Dynasty*, pp. 257–295. [Chen Yuan 1989] deals more in general with the foreigners in Mongol China and their heritage, whereas [Allsen 1983] treats specifically of the Uighurs. More information on Yelü Chucai can be found in [de Rachewiltz 1993: 136–175]. His report of the western expedition of Chinggis Khan with a polemic against Taoism was translated into English in [de Rachewiltz 1962].

3 More than 200 different *zīj*es in Arabic, Persian and some other languages were written by Muslim astronomers from the 8th to the 19th centuries. The typical topics treated in these works include chronology, trigonometry and spherical astronomy, planetary longitudes and latitudes, eclipses, and mathematical astrology; in many cases we also find tables of geographical coordinates and stellar positions. Nearly all Islamic *zīj*es were based on the geocentric, geometrical models for planetary motion as expounded by Ptolemy in his *Almagest*. The tables in *zīj*es allow the calculation of planetary positions and the prediction of the times and magnitudes of solar and lunar eclipses by means of only very few simple arithmetical operations. The text in *zīj*es consists of instructions for using the tables and, less often, explanations and proofs for the underlying models. [Kennedy 1956] offers a survey of Islamic *zīj*es, an update of which is currently being prepared by the present author. See also [King & Samsó 2001] and the article "ZĪDJ" in the *Encyclopaedia of Islam, new edition*.

4 Since we have no further information about the Muslim astronomers in the service of Möngke, this item is not treated in more detail in the following section. The full name of Jamāl al-Dīn is mentioned by the historian Rashīd al-Dīn; see also Chapter II, Section 10 of [Yamada 1980, in Japanese].

- In 1280 Guo Shoujing, one of the most famous Chinese scholars from the Yuan and Ming periods, completed the *Shoushili*, the new official astronomical system of the Mongol dynasty. Thanks to the presence of the Islamic Astronomical Bureau, it can be assumed that Guo Shoujing had access to Arabic and Persian sources, and Islamic influence has been suspected in the *Shoushili* as well as in the instruments built by him.

In the following section, the above-mentioned cases of scientific contact between Iranians and Chinese in the Mongol period will be discussed in more detail. Some methods for investigating the relationships between astronomical tables are described in the Appendix.

3. Contacts Between Iranian and Chinese Astronomers

In this section details will be given of the exchange of astronomical knowledge between Iranians and Chinese during the Mongol period. Explicit information on the relationships between the Islamic and Chinese astronomical tables and instruments that may have been involved in these exchanges can only rarely be found in astronomical or historical sources. Therefore a comparison of the technical characteristics of tables and instruments is indispensible. In particular for the analysis of the Chinese translation of an Islamic *zīj* called *Huihuili* (see Section 3.3 below), the methods described in the Appendix for investigating the relationships between astronomical tables were extensively used. References to publications giving more details of such analyses are also provided. In each case the history of the Mongol empire in the 13th century provides essential background information.

3.1 Yelü Chucai's Adjustment of the Calendar of the Jin Dynasty. When the Mongols conquered Beijing, the capital of Jin, in 1215 they also took over the official astronomical system of that dynasty, the *Revised Damingli* (lit. "Great Enlightenment System"). A Chinese astronomical system or "calendar," as the character *li* 曆 is also often translated, is a set of algorithms for calculating the positions of the sun, moon and planets, the times and magnitudes of eclipses, and various other astronomical quantities. Every Chinese dynasty since the Han (206 BC–AD 221) and, especially in later times, also many individual emperors promulgated their own official astronomical system which was compiled by the astronomers at the imperial Astronomical Bureau.

During Chinggis Khan's long expedition to Transoxiana around the year 1220, his advisor and astrologer / astronomer Yelü Chucai introduced a novelty for Chinese calendars by adjusting the calculations in the *Revised Damingli* for the difference in geographical longitude between mainland China and Samarkand. Sun Xiaochun [1998, in Chinese] has recently argued that he did this under the influence of Islamic *zīj*es. It is in fact known from Chinese sources that Yelü Chucai became familiar with Islamic astronomy during his stay in Samarkand and that he highly appreciated the accuracy of the predictions of eclipses that were possible with tables from *zīj*es.

According to one source he even wrote an astronomical work based on Ptolemaic methods himself, which was referred to as *Madabali* 麻答把曆, but is not extant.

Most Islamic as well as Chinese astronomical tables are designed for a specific locality, producing planetary positions and times and magnitudes of solar and lunar eclipses that are correct only for that locality. For instance, in the case of China the results of the calculations were valid for the capital only, since all official astronomical activity took place at the imperial court. The adjustment to a locality with a different geographical longitude is nothing more than the correction for the difference in local time. (Local time is defined in such a way that the so-called "mean sun," which moves with the same average velocity as the true sun but on the equator instead of on the ecliptic and with a uniform angular velocity instead of a variable one, always culminates precisely at noon.) Thus the planetary position calculated for noon at the base locality of a *zīj* is the position at 11 am at a locality 15° further west and at 1:30 pm at a locality 22°30′ further east. Similarly, if a lunar eclipse is predicted to begin at 3:30 am at the base locality of a *zīj*, it will begin at 2:30 am at a locality 15° further west and at 5 am at a locality 22°30′ further east.

The simplest way to adjust the planetary positions obtained from a *zīj* to a different geographical longitude, is to modify the mean positions. These are the linear functions of time from which the actual positions are found by applying a non-linear correction consisting of one or two so-called "equations." For each 15° west of the base locality, the mean motion in one hour should be added to the mean position at the base locality (since local noon occurs one hour later); for each 15° east of the base locality, the mean motion in one hour should be subtracted. After the adjusted mean position has been determined, the non-linear correction is applied in precisely the same way as before.

Many Islamic *zīj*es provide with each mean motion table a small table for "the difference between the two longitudes" (*mā bayn al-ṭūlayn*) indicating for each geographical longitude the correction to be applied. If μ is the daily mean motion of a given planet and λ_0 the longitude of the base locality, then for each longitude λ the tabulated value Δ is given by

$$\Delta(\lambda) = \frac{|\lambda - \lambda_0|}{360^\circ}\mu,$$

to be added to the mean position found from the table if $\lambda < \lambda_0$, otherwise subtracted (only incidentally were longitudes measured from a zero meridian in the east, in which case the two conditions should be interchanged).

Instead of mean motions, Chinese calendars make use of period relations and calculate the numbers of days between the beginnings of these periods and certain phenomena such as the winter or summer solstice and new or full moon. Since all quantities involved are expressed in days, the correction for a difference in geographical longitude can also be expressed in days and hence is the same for each calculation.

The reworking of the *Revised Damingli* prepared by Yelü Chucai is entitled *Western Expedition Calendar for the Epoch Year* Gengwu, and is extant as Chapters 56–57 of the *Yuanshi*, the official annals of the Yuan dynasty. The epoch mentioned in the title, the starting point of all calculations of planetary motions in this calendar, corresponds to the year AD 1210. Besides taking Samarkand as its base locality, Yelü Chucai's reworking also introduces a concept called *licha* 里差 (lit. "*li* difference"; during the Song and Yuan dynasties a *li* was equal to 441 m). The *licha* is a correction to be applied for localities different from Samarkand and is calculated as $0.04359 \cdot \Delta L$, where ΔL is the (east-west) distance from Samarkand in *li*s [cf. Sun Xiaochun 1998: 4]. The resulting number is the difference in local time expressed in units of which 5230 equal one day.[5] Thus in order to obtain the time difference in hours, the resulting number must still be divided by $5230/24$. Finally, it is added to the local time at Samarkand for localities further east, and subtracted for localities to the west, to obtain the local time of the desired astronomical phenomenon.

Using information from an explanatory work by Yelü Chucai, it is possible to deduce the geographical data used by him to arrive at the algorithm for the *licha* given above. He writes that at Samarkand he observed a partial lunar eclipse approximately 2.6 hours ahead of the time predicted by the *Revised Damingli* for the former Song capital and important astronomical centre Kaifeng (this time was converted; the actual report uses so-called "watches" of the night). By calculating backwards with the algorithm for the *licha* we find that this corresponds almost exactly to a distance of 13,000 *li*s between Kaifeng and Samarkand. Sun Xiaochun notes that this distance is too large by a factor of roughly 1.4 and suggests a possible relationship with the data in Ptolemy's *Geography* (and hence in Islamic geographical tables), in which the longitude difference between Samarkand and cities in China is also around 1.4 times too large. However, there are various uncertainties in his analysis, such as the precise length of the *li* during the Yuan dynasty and the identification of localities in China in Ptolemy's *Geography*. It is also unclear how the *licha* should be calculated for a locality with a completely different latitude from Samarkand, since it is not explicitly specified in Yelü Chucai's reworking of the *Revised Damingli* that the **east-west** distance in *li*s must be used. Nevertheless, it is safe to conclude that Yelü Chucai was inspired by Islamic examples when he implemented the *licha*; however, in doing so he stayed completely within the traditional Chinese framework.

3.2 The Chinese-Uighur Calendar. Hülegü Khan made Maragha in northwestern Iran the capital of his newly founded Ilkhanate. In 1259, on the instigation of the famous polymath Naṣīr al-Dīn al-Ṭūsī, he had an astronomical observatory built on a hill just outside of the city. A comprehensive observational program was planned to last at least 12 years (the time that Jupiter takes to complete one revolution around the sun), but the *Īlkhānī Zīj* completed by al-Ṭūsī shortly after 1270 did not yet contain the results of these observations, which were only incorporated in a later work by Muḥyī al-Dīn al-Maghribī.[6]

5 The use of such units is a characteristic of Chinese calendars. The base periods, such as the solar year and the lunar month, are all expressed as fractions with a common denominator, which is called *rifa* 日法, i.e., "day divisor," in this case 5230. The day divisor is found by solving a set of linear congruence relations and is typical for each calendar.

6 Information on many aspects of the observatory in Maragha (construction, instruments, astronomers and their works, financial administration, instruction) can be found in [Sayılı 1960,

From references by a contemporary astronomer and a somewhat later historian we know that a Chinese scholar Fu Mengchi or Fu Muzhai was active at the observatory in Maragha.[7] It seems probable that he was the main source for the information on the so-called "Chinese-Uighur calendar" that was included in many Persian *zīj*es from the Mongol period. This calendar was used by the Ilkhans for almost a century and has left traces on modern Iranian almanacs in the form of the use of the Chinese duodecimal animal cycle. The Chinese-Uighur calendar was a lunisolar calendar of standard Chinese type with some elements that were more commonly found in unofficial Chinese calendars. Below follows a brief description of its characteristics with an attempt to trace these back to particular Chinese calendars. A complete description of the technical treatment of the Chinese-Uighur calendar in the *Īlkhānī Zīj* can be found in [van Dalen, Kennedy & Saiyid 1997]; its use in Iranian historical sources from the Mongol period is discussed in [Melville 1994].

The Chinese-Uighur calendar has its basic characteristics in common with the official Chinese calendars of the Song and Jin dynasties (11th to 13th centuries). As in every Chinese calendar, each month starts with the day of new moon and hence lasts 29 or 30 days. In determining the day of new moon, first the time of the *mean* new moon is calculated on the basis of the average length of the lunar month; in the Chinese-Uighur calendar this length is taken to be 29.5306 days. To obtain the time of the *true* new moon, a correction has to be applied to the time of the mean new moon; in the Chinese-Uighur calendar this correction consists of two periodic components, the solar equation and the lunar equation, with maximum values of 0.1840 days and 0.3844 days respectively. The period of the solar equation is equal to the solar year (see below), but that of the lunar equation, called the anomalistic month, is somewhat smaller than a lunar month, namely 27.5546 days. The names of the months are given by means of Turkish numerals as well as in transliterations of the Chinese.

The solar year in the Chinese-Uighur calendar starts with the passage of the sun through the midpoint of the zodiacal sign Aquarius and is of length 365.2436 days. It is divided into 24 equal parts, the so-called *qi* 氣, whose transliterated Chinese names are given in the *Īlkhānī Zīj* and other Persian works. The beginning of a Chinese-Uighur year (i.e., the actual lunisolar year) is the day of the (true) new moon immediately preceding the entrance of the sun into the sign Pisces. (This implies that the beginning of the lunisolar year precedes the beginning of the solar year in approximately half of the cases.) An ordinary year consists of twelve lunar months (354 or 355 days), but to stay in pace with the solar year a leap month is inserted every second or third year (close to seven times in each period of nineteen

Chapter 6]. The description by Muʾayyad al-Dīn al-ʿUrḍī of the instruments available at the observatory was translated into German by Seemann [1928]. An extant notebook by al-Maghribī lists some of the observations made at Maragha and shows how new planetary parameters were derived from them; see [Saliba 1983].

7 The Chinese name in Persian transliteration is mentioned by al-ʿUrḍī in a Tehran manuscript of the work indicated in footnote 6 which was not used by Seemann, and the historian Banākatī, who adds the honorific *Sīng sīng* (for Chinese *xiansheng* 先生, "professor," translated as *ʿārif* "sage"); see [Boyle 1963: 253, n. 4].

years). The place of the leap month within a Chinese-Uighur leap year is not fixed; it is the month that contains the initial point of only one of the 24 divisions of the solar year, whereas all other months contain the beginnings of two such divisions.

Chinese calendars traditionally make use of a Grand Conjunction epoch (Chinese: *shangyuan* 上元) in the far past. In the case of the Chinese-Uighur calendar this epoch is said to fall 88,639,679 years before the accession to the throne by Chinggis Khan in AD 1203. The years since the epoch are reckoned in *wan* 萬, i.e., tenthousands. However, different from many official Chinese calendars, all practical calculations in the Chinese-Uighur calendar are carried out on the basis of a sexagesimal cycle of years (the cycle that is mostly used in the *Īlkhānī Zīj* starts in the year 1264). Note that the Chinese sexagesimal cycle was obtained by combining the duodecimal cycle of earthly branches or animals with the decimal cycle of heavenly stems. It was also used for counting the days in a way similar to the use of the days of the week.

Finally, the descriptions of the Chinese-Uighur calendar in Persian sources state that a day was divided into 12 double-hours, the first of which started at 11 pm, and that each double-hour was further divided into 8 quarters (Chinese *ke* 刻; traditionally a *ke* was defined as a hundredth of a day and hence slightly shorter than the *ke* of 15 minutes used in the Chinese-Uighur calendar).

On the basis of the characteristics of the Chinese-Uighur calendar listed above we may now try to draw conclusions concerning its origin. The general characteristics, such as the beginning of the month and year, the use of a correction to determine the true new moon from the mean new moon, and the insertion of the leap month, are common to most contemporary Chinese calendars. The length of the lunar month is correct to tenthousandths; most Chinese and Islamic astronomical works give this parameter with a higher precision. The lengths of the anomalistic month and the solar year agree with values found in various official Chinese calendars from the Song and Jin dynasties (11th to 13th centuries). The only characteristic that can with certainty be associated with one particular Chinese calendar is the Grand Conjunction epoch, which agrees precisely with the epoch of the *Revised Damingli* of the Jin dynasty, but not with any other known calendar. It thus seems probable that the Chinese-Uighur calendar as described in the *Īlkhānī Zīj* was dependent on the *Revised Damingli*.

However, the Chinese-Uighur calendar also has a number of characteristics that are rather atypical for official Chinese calendars. These are: the use of tenthousandths of a day instead of values to a larger number of decimal places or fractions with a common denominator (cf. footnote 5); the use of parabolas for the solar and lunar equations instead of linear or quadratic interpolation between observed values; the use of the Babylonian approximate value $\frac{248}{9} \approx 27.5556$ days for the length of the anomalistic month in the calculation of the lunar equation; the use of a recent epoch for all practical computations instead of the Grand Conjunction epoch. Such characteristics are more typical for the unofficial Chinese calendars that were made by various private scholars not connected to the imperial Astronomical Bureau.

Most unofficial Chinese calendars are lost, but there is one calendar in particular that is known to have had all four characteristics mentioned above. This was the

Futianli 符天曆 ("Heavenly Agreement System") compiled by Cao Zhiwei 曹士蔿 around the year 780 (see [Yabuuti 1982], in Japanese). It was used for the astronomy examinations at the Imperial Academy for several centuries and is extant in a small fragment in Nara (Japan). Some Chinese sources indicate that Cao Zhiwei lived in the western part of China, i.e., in or near the Uighur empire that flourished in the same period. It is therefore tempting to conjecture that the above-mentioned characteristics of the Chinese-Uighur calendar that stem from unofficial Chinese calendars derive from the original calendar of the Uighurs. In that case the Chinese-Uighur calendar as described in Persian *zīj*es from the Mongol period would be a mixture of the *Revised Damingli* of the Jin dynasty, adopted by the Mongols when they captured Beijing in 1215, and the calendar of the Uighurs, which undoubtedly came to play a role in the administration of the Mongols at the time when the Uighurs entered their service around the year 1208.

3.3 The Huihuili ("*Islamic Astronomical System*"). In 1271 the Mongol Great Khan Khubilai founded an Islamic Astronomical Bureau in the Yuan capital of Beijing and appointed the Muslim scholar Zhamaluding 札馬魯丁 as its first director. Presumably this Zhamaluding was the same person as the Jamāl al-Dīn Muḥammad ibn Ṭāhir ibn Muḥammad al-Zaydī from Bukhara who, according to the famous historian Rashīd al-Dīn, served Khubilai's brother Möngke in the Mongol capital Karakorum in the 1250s. In 1267 Zhamaluding had presented to Khubilai Khan a *zīj* as well as models or diagrams of seven astronomical instruments of Islamic type (cf. Section 3.4 below). The Islamic Astronomical Bureau operated parallel to the Chinese Bureau and had a staff of around 40 people including scholars, teachers and administrative personnel.[8] It is known that the Bureau remained in existence until the early Qing dynasty (17th century), but there are very few direct records of its activities. However, the extent of the observational program that was carried out at the Bureau in the early Yuan dynasty can be judged indirectly from two sources dating from the last third of the 14th century, namely:

- The *Huihuili* 回回曆 (lit. "Islamic Astronomical System"), a Chinese translation prepared in Nanjing in AD 1383 of a Persian *zīj* that was available at the Islamic Astronomical Bureau in Beijing at the time when the Yuan dynasty was defeated by its successor, the Ming (1368–1644). The original version of the *Huihuili* does not seem to be extant. A restoration made in the year 1477 by Bei Lin 貝琳, vice-director of the Astronomical Bureau of the Ming dynasty in Nanjing, is available in the National Library of China in Beijing (without tables) and in the National Archives of Japan in Tokyo (complete); it is nowadays easily accessible in the facsimile edition of the *Sikuquanshu*, the enormous collection of literary works produced under the Qing emperor Qian Long in the late 18th century. The copy of the *Huihuili* in the *Mingshi*, the official historical annals of the Ming dynasty, is an abridged version of Bei Lin's restoration which lacks

8 More information on the Islamic Astronomical Bureau of the Yuan dynasty can be found in [Yabuuti 1954, 1987, 1997; Yamada 1980 (in Japanese)]. The primary sources availabe for an investigation of the achievements of the Muslim astronomers in Yuan China are discussed in more detail in [van Dalen 2002].

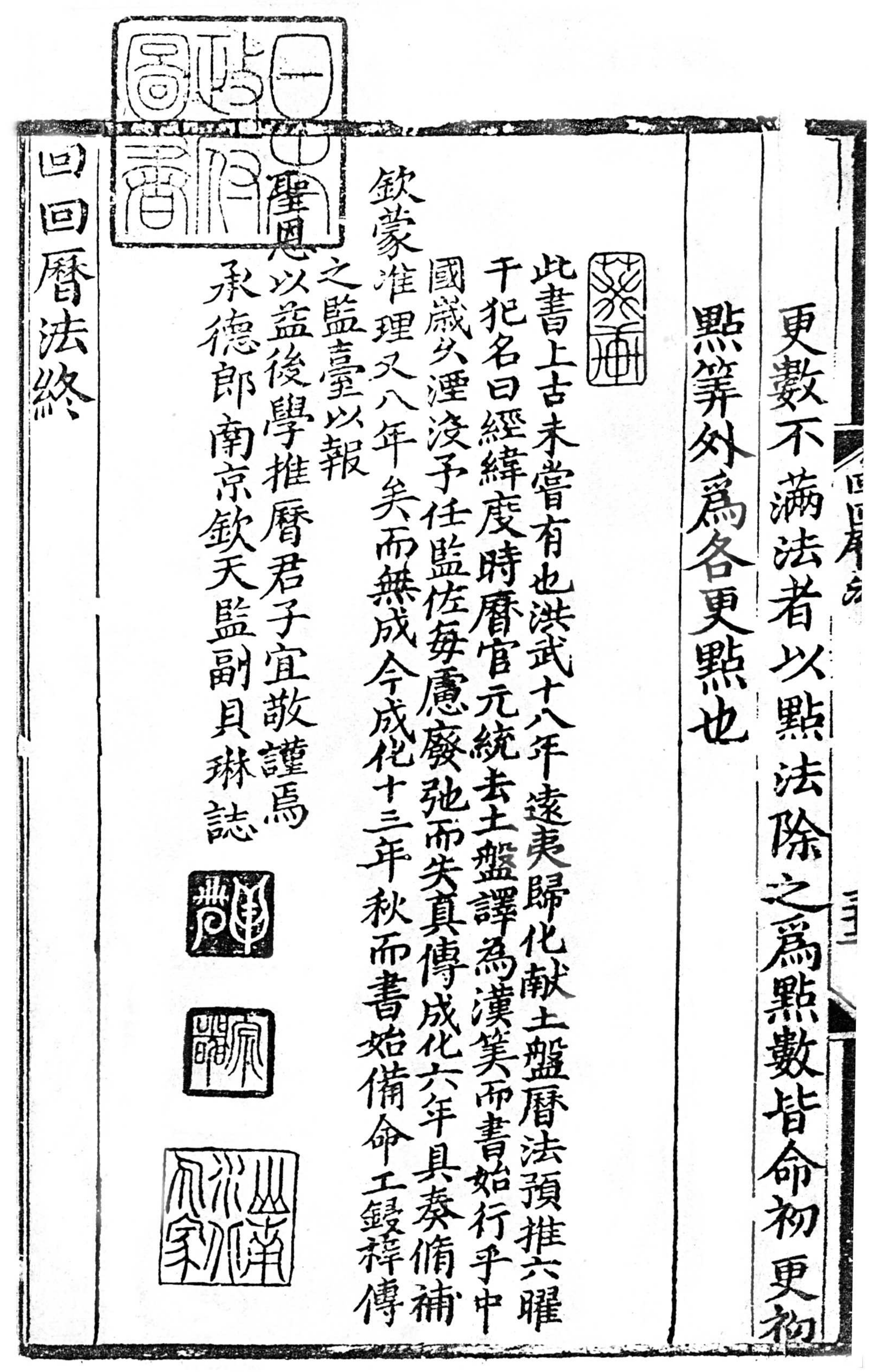

Plate 1. Colophon of the copy of the *Huihuili* in the National Archives of Japan, Cabinet Library (Naikaku Bunko) in Tokyo.

various important tables. More useful is a Korean reworking of the original translation that was prepared on the order of King Sejong in 1442 and contains accurate copies of the tables, whereas the text was adjusted for use in Seoul.[9]

- The *Sanjufīnī Zīj*, the Arabic astronomical handbook by a certain al-Sanjufīnī, written in 1366 for the Mongol viceroy of Tibet. This work is extant in the unique manuscript arabe 6040 of the Bibliothèque Nationale de France in Paris. It contains librarian's notes in Chinese, Tibetan transliterations of month-names, and Mongolian translations of the titles of tables.[10]

I have applied various of the methods described in the Appendix to investigate the relationship between the *Huihuili* and the *Sanjufīnī Zīj*. In the first place, it turned out that almost twenty tables, mostly intended for the determination of planetary positions, are identical in the two works. Of some other tables that have a completely different structure in the two sources, it can be shown that they are based on the same planetary parameters: in the case of the planetary mean motions, the tables in the *Sanjufīnī Zīj* were derived from those in the *Huihuili*, whereas for the planetary latitudes it is the other way around.[11] We can thus conclude that the *Huihuili* and the *Sanjufīnī Zīj* had a common ancestor.

A comparison of the planetary parameters underlying the tables in the *Huihuili* and the *Sanjufīnī Zīj* with those from Arabic and Persian astronomical works included in a parameter file started by E.S. Kennedy (cf. the Appendix), showed that the former do not occur in any other known *zīj*, and in particular not in the *zīj*es resulting from the contemporary activities at the observatory in Maragha, namely the *Īlkhānī Zīj* by Naṣīr al-Dīn al-Ṭūsī and the *Adwār al-anwār* by Muḥyī 'l-Dīn al-Maghribī. Also the setup of various tables in the *Huihuili* and the *Sanjufīnī Zīj* is different from what we find in other *zīj*es. We may therefore conclude that the common ancestor of the two works constitutes a highly original compilation based on an extensive observational program with determination of new planetary parameters and various innovations in the presentation of the tabular material.

That this compilation was produced at the Islamic Astronomical Bureau of the Yuan dynasty is plausible for a number of reasons. Firstly, both the *Huihuili* and the *Sanjufīnī Zīj* were written in China and, as was remarked above, we know of no Islamic astronomical work from outside of China that is based on the same parameters. Secondly, the tables for planetary mean motion in the *Sanjufīnī Zīj* are said to be based on the "observations of Jamāl," which may refer to Zhamaluding.

9 An edition and translation of the text of the *Huihuili* with commentary and transcription of the tables is currently being prepared by the present author. The technical contents of the work were outlined in [Yabuuti 1997] and discussed in some more detail in [Chen Jiujin 1996, in Chinese]. The Korean version of the *Huihuili* was studied by Shi Yunli (to appear in *Archive for History of Exact Sciences*).

10 Philological and historical aspects of the *Sanjufīnī Zīj* were explored in [Franke 1988]; the technical material on eclipses, parallax and lunar visibility was analysed in [Kennedy 1987/88; Kennedy & Hogendijk 1988].

11 The tables for planetary latitude in the *Huihuili* were described and compared with the tables in other *zīj*es in [Yano 1999; van Dalen 1999]. The table for Mercury's equation of centre discussed in the Appendix is investigated in [Yano 2002].

Finally, the Oriental Institute in St. Petersburg is in the possession of a Persian astronomical manuscript with tables numerically identical to those in the *Huihuili*. This manuscript had been obtained in China and was described by A. Wagner [1882] when it was still in the library of the Pulkovo Observatory near St. Petersburg. On paleographical grounds it has been variously dated between the 12th and 13th centuries, and a preliminary investigation indicates that it was a working copy for the Chinese translation prepared in 1383. It contains tables for Beijing that must have stemmed from the original work and were not included in the translation, as well as tables for Nanjing that must have been additions by the translators.

Although the *Huihuili* and the *Sanjufīnī Zīj* are standard Islamic *zīj*es based on Ptolemy's geometrical models for planetary motion, they both show distinguishable Chinese influences. In the case of the *Huihuili* these influences probably stem from the translators in the early Ming dynasty and can be recognized in the following topics:

- The chronological material at the beginning of the work was reformulated with the use of Chinese terminology and methods. Thus the number of years since epoch is called "accumulated years" (*jinian* 積年) and all mean positions are first determined for the vernal equinox rather than directly for the desired date in the Islamic Hijra calendar. Also a rule for the determination of the leap month in the Chinese calendar and the confusion of lunar and solar years that led to the use of a Hijra epoch in AD 599 instead of the correct AD 622 must have originated with the Ming translators.
- The star catalogue in the *Huihuili* lists ecliptical longitudes and latitudes as well as magnitudes for 277 fixed stars. The catalogue is of particular interest, since it is one of only two larger Islamic star tables that were based on new observations rather than having been derived directly from the star catalogue in Ptolemy's *Almagest* (cf. [van Dalen 2000]). Furthermore, it is the earliest table to give a correspondence between Ptolemaic and Chinese star-names. The star table in the *Huihuili* is mainly used for the calculation of so-called "encroachments" (*lingfan* 凌犯), passages of the moon and the planets through stellar constellations, an important topic in Chinese astrology. Whereas traditionally encroachments were observed in order to be interpreted as omens, the *Huihuili* provides the additional possibility of calculating them.

In the case of the *Sanjufīnī Zīj* the Chinese influences must have been part of the original work, since the only extant manuscript is an autograph. They include:

- The use of an epoch named after Chinggis Khan, namely the vernal equinox of the year AD 1207.
- A table of the 24 equal divisions of the solar year, the so-called *qi* (cf. Section 3.2 above), with Persian transliterations of their Chinese names.
- A table of the Arabic lunar mansions with longitudes and latitudes for some of the individual stars and Persian transliterations of the names of the corresponding Chinese mansions. In one or two cases the listed stars are not the standard Arabic ones and may have been influenced by Chinese constellations (cf. [van Dalen 2000: 150–151]).

3.4 Guo Shoujing's Astronomical Instruments and the Shoushili. In 1276, the famous scholar Guo Shoujing, who had previously been involved in both canal and irrigation projects, was assigned to the task of devising a new official astronomical system for the Mongolian Yuan dynasty. For this purpose he first designed a large number of astronomical instruments and carried out systematic observations of winter and summer solstices, positions of planets and stars, etc. In 1280 the new calendar was finally completed and starting from 1281 it was distributed under the name *Shoushili* (lit. "Season-Granting System"). It was by far the most accurate calendar in the history of Chinese mathematical astronomy and continued to be used for almost 400 years (the calendar of the Ming dynasty was called *Datongli*, lit. "Great Concordance System," but differed only insignificantly from the *Shoushili*).

There has been a continuing debate about to what extent Guo Shoujing and his colleagues were influenced by Islamic astronomy. Thanks to the presence of the Islamic Astronomical Bureau in the Yuan capital, the official Chinese astronomers must have had ample opportunity to familiarize themselves with Islamic astronomical instruments and tables. It seems possible that Guo Shoujing consulted Zhamaluding, the director of the Islamic Bureau, and other Muslim astronomers in person. Moreover, eleven Islamic instruments and many Arabic and Persian books that were present at the Bureau were described in official Chinese sources. Seven of these instruments were presented by Zhamaluding to Khubilai Khan in 1267; four more were kept by him at his residence. Among them were various typically Islamic instruments, such as a parallactic ruler, a terrestrial globe, a plane and a spherical astrolabe with ecliptic rings, and a compass. Among the books whose transliterated Persian titles are listed in Chinese annals, we find Ptolemy's *Almagest*, Euclid's *Elements*, al-Ṣūfī's *Constellations of the Stars*, Kūshyār's *Introduction to Astrology*, one or more *zīj*es, and works on topics such as cosmology (*haʾya*), the construction of instruments, chronology, geometry, arithmetic, and technical devices (*ḥiyal*).[12]

However, there is hardly any information in Chinese sources about the extent to which Chinese astronomers made actual use of the available Islamic knowledge. It is known that during the Qing dynasty there was a harsh competition between the Islamic and the Chinese Astronomical Bureaus (some articles about this topic were published in Chinese by Huang Yilong). For the Yuan and Ming dynasties we have to resort to investigations of the extant calendar works and descriptions of instruments or, in some cases, the instruments of Guo Shoujing themselves. A comprehensive analysis of these sources is beyond the scope of this article; instead I will limit myself to some interesting points put forward, in particular, by Needham, Yabuuti and Miyajima. More information can be found in the works mentioned in footnote 12 and [Yamada 1980]. Recent researches on the *Shoushili* include those by Jing Bing and Wang Rongbin published in Chinese.

A frequently quoted statement concerning the influence of Islamic astronomy on the work of Guo Shoujing is found in the introduction of [Sédillot 1847], the

12 The transliterations of Persian names of instruments, book titles, month-names and the days of the week that are found in Chinese sources, were discussed extensively in [Tasaka 1957]. [Hartner 1950; Miyajima 1982] dealt specifically with the names of the instruments presented to Khubilai Khan by Zhamaluding.

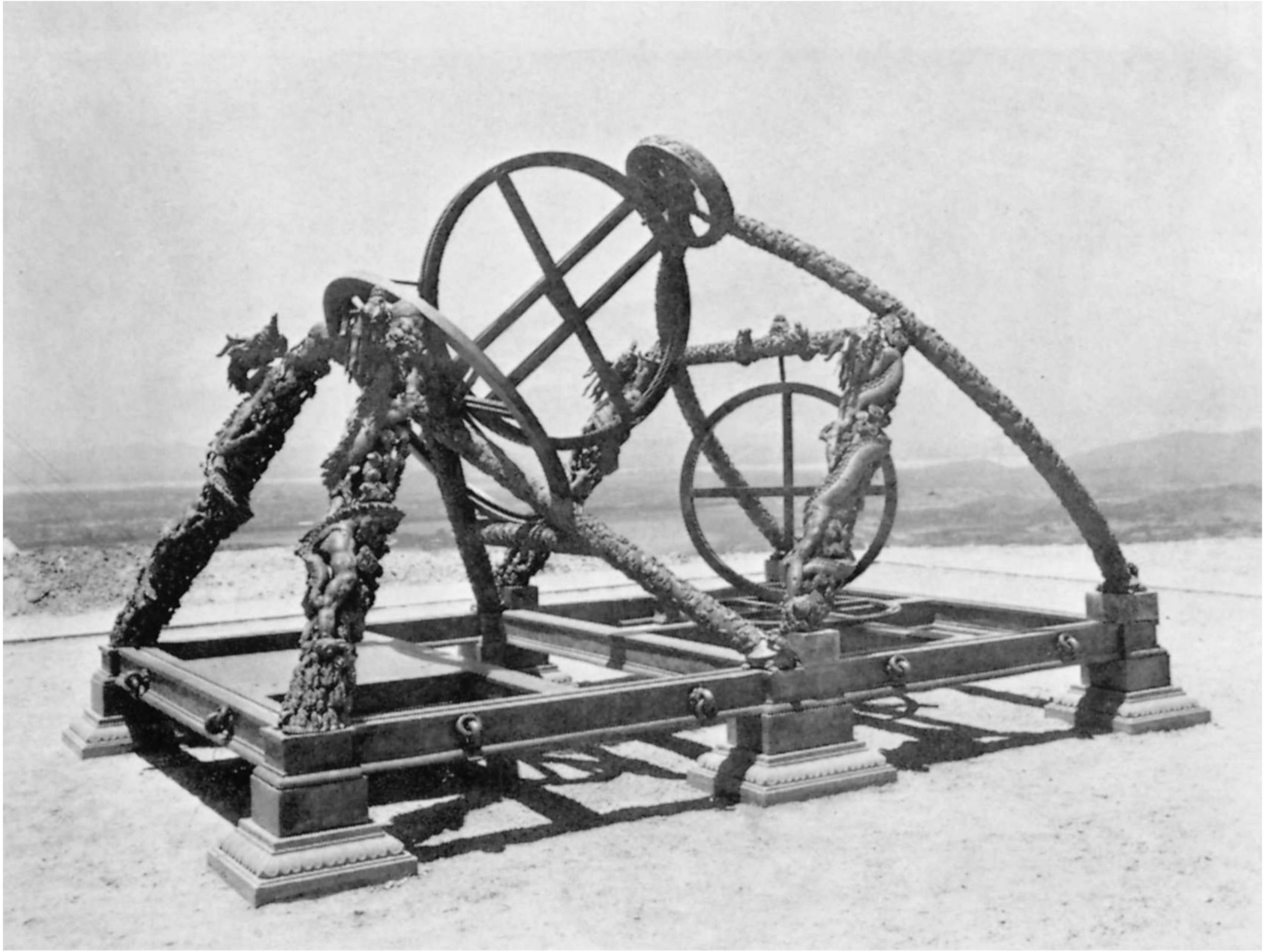

Plate 2. The Simplified Instrument of Guo Shoujing at the Purple Mountain Observatory outside of Nanjing (photograph taken from K. Yamada, *Juji-reki no michi*, Tokyo 1980).

edition of the Persian text of the *zīj* of Ulugh Beg. On page *ci*, Sédillot states that "in 1280 Guo Shoujing received the *zīj* of Ibn Yūnus from Zhamaluding and studied it in detail." In [Sédillot 1845–1849, 2: 484, 640, 642], he adds that Guo Shoujing wrote the *Shoushili* together with Zhamaluding, that Arabic scientific treatises were translated into Chinese at the time, and that Guo Shoujing was the first Chinese to study spherical astronomy. As a matter of fact, not a single of these statements can be proved and most of them are simply wrong; they are all distorted versions of translated passages from Chinese historical annals and their interpretations as presented in [Souciet 1729–32].

In the 20th century more reliable studies concerning the interaction between Islamic and Chinese scholars in the Yuan and Ming dynasties were made and the most important results were summarized in [Needham & Wang 1959: 294–302, 367–382; Yabuuti 1997: 14–17]. As far as the instruments of Guo Shoujing are concerned, both authors give room to the possibility that they were to some extent influenced by Islamic instruments. For instance, they assume that the so-called "Tower of the Duke of Zhou," a masonry tower in Gaocheng near Luoyang in the province Henan with a shadow scale and 40 ft. gnomon which is now lost, was influenced by the giant instruments built by, e.g., al-Khujandī (d. AD 1000) near Rayy in Iran. Furthermore, the 6-meter "Simplified Instrument" (*jianyi* 簡儀, see Plate 2) with its equatorial alignment, projections of the heavenly circles onto three different planes and alidades instead of sighting tubes, has similarities to the torquetum, whose invention has been attributed to Jābir ibn Aflaḥ (12th-century Spain, cf. [Lorch 1976]; note, however, that the torquetum was not among the instruments used in Maragha or those brought to China by Zhamaluding). Finally, the "Upward-Looking Instrument" (*yangyi* 仰儀), which consists of a concave hemisphere with a grid and a pointer and was not known in pre-Mongol China, is very similar to the scaphe sundial that had been common in ancient Greece and the Islamic world and is still found among the Islamic-style instruments built by Jai Singh in 18th-century India. Thus in each of these three cases there is a possibility of Islamic influence.

Recently, in his detailed study of the descriptions in the Yuan annals of the instruments brought to China by Zhamaluding, Miyajima [1982, in Japanese] put the Islamic influence on Guo Shoujing into perspective by noting that, in spite of their similar appearance, the usage of the Islamic and Chinese instruments mentioned above is in fact quite different. Thus the Tower of the Duke of Zhou measures the shadow cast by a gnomon (i.e., wooden post) on a horizontal plate, whereas with the giant Islamic instruments like that of al-Khujandī the sunlight falls through an aperture onto a scale on a concave circular arc. Furthermore, except for the equatorial alignment of the Simplified Instrument, its structure and its scales are quite different from that of a torquetum. Finally, the Upward-Looking Instrument of Guo Shoujing was used for measuring positions on the heavenly sphere, whereas the Greek and Islamic scaphe sundials were designed to measure the time of day.

It is also difficult to point out precisely the Islamic influence in Guo Shoujing's astronomical system, the *Shoushili*.[13] The use of an epoch in the recent past instead of a Grand Conjunction epoch millions of years ago, as well as the consistent use of decimal numbers in all parameters and calculations have been related to Muslim practices, but are also found in unofficial Chinese calendars (besides, Islamic astronomical works make use of sexagesimals much more than decimals).

The same holds for the use of cubic corrections for the motions of the planets. As was indicated above for the Chinese-Uighur calendar, the actual position of the sun, moon and planets is calculated, both in Ptolemaic and in traditional Chinese astronomy, by applying a periodic correction to a linear function of time. In Chinese calendars before the *Shoushili* these corrections were parabolic functions calculated by means of first-order differences. However, in the *Shoushili* cubic functions were applied; for example, for the sun the correction q is given by:

$$q = 0.051332d - 0.000246d^2 - 0.00000031d^3$$

for days d reckoned from the the winter solstice, and by

$$q = 0.048706d' - 0.000221d'^2 - 0.00000027d'^3$$

13 The canons of the *Shoushili* are contained in Chapters 54–55 of the *Yuanshi*, an evaluation by a government committee in Chapters 52–53. A summary of the technical contents of the work can be found in [Nakayama 1969: 123–150]. I am grateful to Prof. Nathan Sivin for letting me use his unpublished translation of, and commentary on, the *Shoushili*, as well as for his useful comments on a preliminary version of this section.

for days d' reckoned from the summer solstice. Here q is expressed in Chinese degrees equal to a 1/365.25th of a complete apparent rotation of the sun around the earth (i.e., one Chinese degree corresponds to the average daily solar motion). The actual calculations were performed by means of a table with second-order differences of the corrections. Since in Ptolemaic astronomy all corrections to the linear mean motions of the planets were trigonometric functions (e.g., the correction for the sun, the so-called "solar equation," is given by

$$q = \arcsin\left(\frac{e \cdot \sin \bar{a}}{60 + e \cdot \cos \bar{a}}\right),$$

where $\bar{a}$ is a linear function of time and e is the eccentricity of the circle of uniform motion of the sun with respect to the earth), there is once more no clear indication of transmission. It seems rather that the planetary corrections in the *Shoushili* presented a further development of those in earlier Chinese calendars with at most a very subtle influence of Islamic knowledge.

Because of the equatorial character of Chinese astronomy (most measurements were made with respect to the north pole) and the fact that the planets move on or near the ecliptic, conversion of ecliptical coordinates into equatorial ones and vice versa was essential in Chinese planetary theory. Since the Chinese astronomical systems did not make use of any form of geometry and of only little trigonometry, this conversion had to be approximated rather than carried out exactly. In calendars from the Tang dynasty (AD 618–905) this was done by means of a linear step function. During the Song dynasty we find a more refined numerical approximation of parabolic type. Guo Shoujing, finally, made a first step towards a geometric method, although he made use only of plane triangles, not of spherical ones. The resulting formula, based on the "arc-chord-sagitta relationship" formulated by the Song scholar Shen Gua (see [Nakayama 1969: 137–139]), is only in some parts of the equator and ecliptic more accurate than the earlier quadratic method.

It seems possible that Guo Shoujing's step towards geometry in the conversion between ecliptical and equatorial coordinates was influenced by Islamic astronomy. On the other hand, both the actual method of conversion and the implementation of the necessary calculations by means of a table with first order differences fall clearly within the traditional Chinese framework. In no instance in the *Shoushili* do we find ready Islamic results or tables.

4. Conclusion

In this article we have studied various aspects of the transmission of astronomical knowledge between the Islamic world and China under Mongol rule. We have sketched the historical background of the interaction between Muslim and Chinese scholars in the 13th century and have thus made clear that the possibilities for contacts were numerous, and that the transmission took place by means of personal transfer rather than through written works or instruments brought by, for instance, merchants. Whereas it seems that the number of Chinese in Ilkhanid Iran was rather small, in the last third of the 13th century tens of thousands of Iranians, Uighurs and other Muslims were active in Mongol China in administrative functions and also as scholars and artists.

We have investigated four particular cases of transmission in some detail.

- Around the year 1220, Yelü Chucai, advisor of Chinggis Khan, adapted the official Chinese calendar of the Jin dynasty for use in Transoxiana and at arbitrary geographical localities (Section 3.1 above). Yelü is known to have familiarized himself with Islamic astronomy and probably carried out the adaptation under the influence of Islamic *zīj*es.
- The Ilkhans introduced into Iran the so-called “Chinese-Uighur calendar” (Section 3.2). This was a typical Chinese calendar whose elements can be traced back to the calendar of the Jin dynasty as well as to unofficial Chinese calendars. Its descriptions in Persian *zīj*es from the year 1270 onwards presumably stemmed from a Chinese astronomer active at the famous observatory in Maragha.
- In AD 1383 the first emperor of the Ming dynasty had some Persian astronomical works translated into Chinese that were available at the Islamic observatory of the conquered Yuan capital Beijing. These works included Kūshyār’s *Introduction to Astrology* and a *zīj* with tables and parameter values not known from any other Islamic work (Section 3.3). The *zīj* can be assumed to have been an original achievement of the Muslim astronomers active in China around the year 1275 and is based on completely new planetary and stellar observations.
- Guo Shoujing finished the most accurate calendar in the history of Chinese mathematical astronomy, the *Shoushili*, in AD 1280 (Section 3.4). Islamic influence has been suspected in this calendar as well as in Guo’s instruments, but closer inspection reveals that this influence was very minor and that calendar and instruments stand clearly in the Chinese tradition.

The transmissions in each of these four cases were of a very different nature. Yelü Chucai visited the Islamic world himself and must have had ample opportunity to learn Persian and to become acquainted with Islamic astronomy through written works as well as personal contacts (even in cities where the Mongols slaughtered the population, they spared the lives of scholars and artisans). The Chinese astronomer active at the observatory in Maragha seems to have been isolated and was probably the only source of information on the Chinese-Uighur calendar that was available to his Arabic and Iranian colleagues. On the other hand, a relatively large number of Muslim scholars worked at the Islamic Astronomical Bureau of the early Yuan dynasty. They must have brought with them from Iran and other regions many Arabic and Persian works on mathematics and astronomy, but in China they compiled a completely new *zīj* based on an extensive program of observations. The same Muslim astronomers and the works they had brought with them may have influenced Guo Shoujing and his colleagues when they compiled the *Shoushili* and built various new instruments; it seems probable, though, that these contacts had to take place through interpreters and hence would have been quite problematic.

Also as far as the assimilation of the transmitted astronomical knowledge is concerned, there are clear differences between the four cases. Both Yelü Chucai in his adaptation of the Jin calendar and Guo Shoujing in the *Shoushili* made use of some Islamic concepts or ideas such as the adjustment of the times of phenomena to the local geographical longitude and the use of geometrical / trigonometrical methods for the conversion between ecliptical and equatorial coordinates. However, neither Chinese astronomer took over actual algorithms or tables from Islamic *zīj*es and the resulting calendars fitted in almost every respect within the Chinese tradition. The Chinese translation of the *zīj* compiled by the Muslim astronomers in Yuan China basically left the Islamic (i.e., Ptolemaic) character of the work unchanged, although Chinese concepts were introduced to make certain topics easier to understand for Chinese astronomers. In particular the star table with ecliptical coordinates, displaying Ptolemaic as well as Chinese star names, and its application to the typically Chinese problem of "encroachments," may have been the result of a joint effort of Muslim and Chinese astronomers. Similarly, the descriptions of the Chinese-Uighur calendar in Persian *zīj*es contain little Islamic influence. The algorithms are of Chinese type and the technical terminology is given almost exclusively in transliterations of the Chinese. As far as the numerous auxiliary tables are concerned, some of them, such as the solar and lunar equations, may have been present in a Chinese original, whereas most others display simple multiples of the base parameters of the calendar and could have been added by the Muslim authors.

Some mathematical methods for investigating the relationships between astronomical tables are described in an Appendix to this article. Since the numerical data in such tables are highly idiosyncratic, connections between them can usually be reliably established. In cases where tables from different cultures and / or periods can be shown to be connected, we may have found a case of transmission. The situation is much more difficult for computational algorithms and instruments. In various examples discussed briefly in this article (in particular, the calendar and instruments of Guo Shoujing) it is clear that at least complete descriptions of structure as well as usage should be considered in order to be able to make well-founded statements concerning possible connections.

Appendix: Methods for Investigating Relationships Between Astronomical Tables

In many cases mathematical tables in manuscripts of *zīj*es are not clearly attributed to a particular astronomer. Therefore, in my analyses of the connections between astronomical works from different periods and / or geographical regions I have made an extensive use of comparisons of numerical data in tables. Such comparisons have turned out to be very effective for investigating relationships, because the data are generally exact and highly idiosyncratic (two tables for the same function need only have relatively small differences to be able to conclude that they stem from two different sources). Since data in astronomical tables generally show obvious regularities (usually because of the continuity of the tabulated functions), incidental scribal errors can often be reliably corrected. Systematic differences between two tables for the same function can have a number of causes, such as the use of different values for the underlying astronomical parameters or the use of different computational techniques (e.g., approximate algorithms instead of the "exact formula," computation from inaccurate auxiliary tables, use of linear or quadratic interpolation in auxiliary tables, the rounding of intermediate results to a certain number of digits, etc.). Incidental differences between two tables for the same function that cannot be explained as scribal errors may be due to computational mistakes.

Three examples of methods that can be used to investigate the relationships between astronomical tables are presented below. Statistical or numerical tools for performing more sophisticated tasks, such as estimating the parameter values underlying an astronomical table and testing the dependence of two tables for the same or for different functions, are not discussed in detail but are referred to where appropriate.

Case 1: Comparison of Complete Tables. The simplest way to investigate the relationship between two tables for a particular astronomical function is to compare them value by value. If all values are identical, we may assume that the tables ultimately derive from the same source. However, an exception to this rule is a pair of tables without errors, since we can hardly ever exclude the possibility that two astronomers independently calculated a correct table for a given function. (In this context the error in a tabular value is defined as the difference between that value and a recomputation based on the modern formula for the tabulated function; a table is correct if it does not have any errors.) Tables without errors can be found among tables of very simple functions such as linear ones, or among tables of more complicated functions with values to only a small number of sexagesimal places (typical examples are sine tables with values to a precision of three sexagesimal places). As was indicated above, scribal errors require a special treatment: if the differences between two tables can all be explained as incidental scribal mistakes, the tables can be considered to be mathematically identical and hence may be related in spite of the differences. If two tables have a very conspicuous set of scribal errors in common, they can be assumed to have been copied from the same erroneous original.

Table 1 shows three fragments of the tables for Mercury's equation of centre in *al-Qānūn al-Masʿūdī* by al-Bīrūnī (Afghanistan, ca. 1030) and the *Huihuili* (China, ca. 1275, see above).[14] The equation of centre is one of the six or seven functions that are tabulated in Ptolemy's *Almagest* and in most Islamic *zīj*es to allow the easy calculation of planetary longitudes. It is a highly complicated trigonometric function that can be derived from Ptolemy's geometrical models for the planetary motions (see, for instance, [Neugebauer 1957, Appendix 1] or, for more details, [Pedersen 1974, Chapters 9 and 10]). It is tabulated as a function of the so-called "mean centrum," a linear function of time.

The third and fifth columns of Table 1 show the errors in the tables in *al-Qānūn al-Masʿūdī* and the *Huihuili*, namely, the differences between the values in these

14 Sexagesimal numbers are given in the standard notation, i.e., sexagesimal digits are separated by commas, whereas the sexagesimal point is represented by a semicolon. For instance, 2;15,29 denotes $2 + \frac{15}{60} + \frac{29}{60^2}$.

mean centrum	Bīrūnī	Bīrūnī *minus* recomputation	*Huihuili*	*Huihuili minus* recomputation	Bīrūnī *minus Huihuili*
10	0;31	+5	0;31	+5	
11	0;34	+5	0;34	+5	
12	0;36	+5	0;37	+6	−1
13	0;39	+5	0;40	+6	−1
14	0;42	+6	0;43	+7	−1
15	0;46	+7	0;46	+7	
16	0;49	+7	0;49	+7	
52	2;13	+4	2;13	+4	
53	2;15	+3	2;15	+3	
54	2;14		2;16	+2	−2
55	2;17	+1	2;18	+2	−1
56	2;19	+1	2;19	+1	
57	2;20		2;21	+1	−1
58	2;22		2;22		
59	2;23	−1	2;24		−1
60	2;25	−1	2;25	−1	
61	2;28		2;27	−1	+1
62	2;29		2;29		
63	2;30	−1	2;30	−1	
89	2;43	−18	2;43	−18	
90	2;43	−18	2;43	−18	
91	2;43	−18	2;42	−19	+1
92	2;42	−20	2;42	−20	
93	2;42	−20	2;42	−20	
94	2;42	−20	2;42	−20	
95	2;42	−20	2;42	−20	
96	2;42	−20	2;41	−21	+1
97	2;41	−21	2;41	−21	
98	2;41	−20	2;41	−20	

Table 1. Comparison of the tables for Mercury's equation of centre in al-Bīrūnī's *al-Qānūn al-Masʿūdī* and the *Huihuili*.

two works and a recomputation on the basis of Ptolemy's Mercury model and his parameter value. Since the tables differ systematically from the recomputation by up to 21 minutes of arc, we may conclude that both were calculated according to an algorithm and / or parameter value different from those used by Ptolemy. However, since the two tables differ from each other in only 13 out of 180 values (see the last column), it is highly probable that they were computed on the basis of the same algorithm and parameter value. A statistical test of dependency as described in [Van Brummelen & Butler 1997] may be necessary to determine whether the authors of the *Huihuili* simply copied al-Bīrūnī's table for the first equation of Mercury or calculated it from scratch. Since the differences occur in small groups in which they tend to have the same sign, it seems that the authors of the *Huihuili* carried out at

source	*value*
Ptolemy (Alexandria, ca. 140)	0;59,8,17,13,12,31
Ibn al-Shāṭir (Damascus, ca. 1350)	0;59,8,19,36, 0
Alfonsine Tables (Spain, ca. 1275)	0;59,8,19,37,19,13,56
Ibn Yūnus (Cairo, ca. 1000)	0;59,8,19,42,35
Banū Mūsā ibn Shākir (Baghdad, ca. 840)	0;59,8,19,43,14,18
Naṣīr al-Dīn al-Ṭūsī (Maragha, ca. 1270)	0;59,8,19,43,47
Shams al-Munajjim al-Wābkanwī (Maragha, ca. 1320)	0;59,8,19,43,47
Ibn Yūnus (Cairo, ca. 1000)	0;59,8,19,44,10,30,32, 0
Ghiyāth al-Dīn al-Kāshī (Samarkand, ca. 1420)	0;59,8,19,44,10,39
modern value	0;59,8,19,48,43,18
Huihuili (Beijing, ca. 1275)	0;59,8,19,49,27,22,58
Muḥyī 'l-Dīn al-Maghribī (Maragha, ca. 1280)	0;59,8,20, 8, 4,36,38
ʿAbd al-Raḥmān al-Khāzinī (Marv, ca. 1120)	0;59,8,20,13,18
Yaḥyā ibn Abī Manṣūr (Baghdad, ca. 830)	0;59,8,20,35,14,38
Ḥabash al-Ḥāsib (Baghdad, ca. 830)	0;59,8,20,35,25
al-Bīrūnī (Afghanistan, ca. 1030)	0;59,8,20,38,21,13
Abū 'l-Wafāʾ al-Būzjānī (Baghdad, ca. 975)	0;59,8,20,43,17,38,41,42
al-Battānī (Syria, ca. 900)	0;59,8,20,46,49

Table 2. Values for the daily solar mean motion used
by Ptolemy and some important Muslim astronomers.

least some calculations themselves, e.g., they may have corrected some mistakes in al-Bīrūnī's table by carrying out anew the linear interpolation between certain values for multiples of 10°.[15]

Case 2: Comparison of Parameter Values. Instead of comparing complete tables, one may compare certain mathematical characteristics of tables, such as computational techniques (see Case 3 below) or the underlying values of the astronomical parameters involved. Methods for extracting parameter values from astronomical tables are described in [van Dalen 1989, 1993, 1996; Mielgo 1996]. Although for certain parameters, such as the obliquity of the ecliptic, only very few different values were in use (the value 23°35′ was particularly common among Muslim astronomers), especially parameters measuring planetary mean motions tend to be highly idiosyncratic. Table 2 shows an excerpt from the file collected by Professor E.S. Kennedy with parameters from about 30 important Islamic *zīj*es and several other astronomical works. The values for the daily solar mean motion listed here are generally very close, but the differences are still large enough to be able to distinguish between the results of different observations or determinations. For example, Shams al-Munajjim al-Wābkanwī apparently used al-Ṭūsī's value, and the value of Ghiyāth al-Dīn al-Kāshī seems to be dependent on that of Ibn Yūnus. On the other hand, the value from the *Huihuili* points to an independent observation.

15 [Yano 2002] contains an extensive analysis of the tables for Mercury's equation of centre in the *Qānūn* and the *Huihuili*. Yano showed that al-Bīrūnī's table is based on Ptolemy's parameter value but a different, apparently erroneous formula for calculating the equation of centre.

Case 3: Comparison of Mathematical Techniques. The use of inaccurate auxiliary tables or certain mathematical techniques such as interpolation for the computation of an astronomical table may leave traces in the tabular values in the form of identifiable error patterns. Like the underlying parameter values, the auxiliary tables and computational techniques that were used may be typical for a certain astronomer or school of astronomers and hence may be used to determine the origin of tables or, at least, to establish relationships between them.

In the first example below, it will be shown that the error pattern in a tangent table is tightly connected with the errors in the sine table from which it was calculated. In the case of the sine and tangent tables from the so-called *Baghdādī Zīj* (late 13th century), this helps us to conclude that both tables derive from the important tenth-century astronomer Abū 'l-Wafā᾽ al-Būzjānī, whose *zīj* is non-extant. In the second example, it will be shown how the use of a special type of second-order interpolation can be recognized in the highly accurate sine table of Ulugh Beg (early 15th century). The same type of interpolation also underlies Ulugh Beg's tables of oblique ascensions, but has not been found in the work of any other astronomer.

Example 1. The third column of Table 3 shows the errors for arguments between 60° and 90° in the tangent table found on folios 227v–228v of the *zīj* by a certain Jamāl al-Dīn Abī 'l-Qāsim ibn Maḥfūẓ al-munajjim al-Baghdādī, which was written in 1285 and is extant in the unique manuscript Paris BNF arabe 2486. In my dissertation [van Dalen 1993: 164–168] I have conjectured that this tangent table and a number of other tables in the *Baghdādī Zīj* with values to sexagesimal thirds were taken from the astronomical handbook *al-Majisṭī* by the important 10th-century mathematician and astronomer Abū 'l-Wafā᾽ al-Būzjānī. The text of this work is extant as Paris BNF arabe 2494, but the tables are lost except for scattered fragments. Meanwhile I have worked through the text of *al-Majisṭī* and have found more evidence for my conjecture which I hope to publish in due course. In the remainder of this article, when speaking of the "sine and tangent tables of Abū 'l-Wafā᾽," I mean the tables from the *Baghdādī Zīj*

The tangent function tabulated by Abū 'l-Wafā᾽ and most other Muslim astronomers was $60 \cdot \tan x$ rather than $\tan x$. As can be seen clearly from the third column of Table 3, which displays in sexagesimal thirds the differences between Abū 'l-Wafā᾽'s table and exact tangent values, the errors in the tabular values increase rapidly as the argument x approaches 90°. This is typical for many historical tangent tables and can be explained as follows. The tangent is calculated according to

$$\tan x = \frac{\sin x}{\cos x}.$$

When the argument x approaches 90°, the denominator in this expression, $\cos x$, nears zero. Therefore, if it is consistently rounded to the same number of sexagesimal places, its relative error (i.e., the rounding error divided by the cosine itself) increases. Since the relative error in the tangent is of the same order of magnitude as that in the cosine, it also increases when the argument approaches 90°. Finally, since the tangent itself becomes very large towards 90°, the absolute error in the calculated

arc	tangent in the *Baghdādī Zīj*	*Baghdādī Zīj* minus recomputation	*Baghdādī Zīj* minus reconstruction
60	103;55,22,58		
61	108;14,34,17	−2	
62	112;50,36,56	+1	−1
63	117;45,23,52		−1
64	123; 1, 5,38		
65	128;40,13,33	+3	
66	134;45,43,54	−3	−2
67	141;21, 4,12	+5	
68	148;30,18,42	−4	−1
69	156;18,19,10	−4	−1
70	164;50,55, 6	−1	−1
71	174;15, 9,27	−6	
72	184;39,39,40	+1	
73	196;15, 4,15	+5	+1
74	209;14,41,34	+3	
75	223;55,22,57	−1	
76	240;38,48,39	−2	
77	259;53,18,41	−6	
78	282;16,39,50	−16	−1
79	308;40,23,27	−13	
80	340;16,37, 8	+16	
81	378;49,30, 6	−14	−1
82	426;55,20,10	+18	
83	488;39,38,28	−22	
84	570;51,42,35	−8	
85	685;48,12,11	+53	+1
86	858; 2,23,33	−22	
87	1144;52, 5,34	+3	+1
88	1718;10,37,11	+388	
89	3437;24, 1,51	+608	

Table 3. Recomputation of the tangent table of Abū ’l-Wafāʾ as contained in the *Baghdādī Zīj* with a reconstruction from his sine table.

tangent, as tabulated in the third column of Table 3, grows even more rapidly than the relative error.

The fourth column of Table 3 displays the differences between Abū ’l-Wafāʾ’s tangent table and values reconstructed by applying the formula

$$\tan x = \frac{\sin x}{\sin(90^\circ - x)}$$

to values from his sine table. This sine table is also included in the *Baghdādī Zīj* (folios 224v–225v) and contains only nine errors of one sexagesimal third. The agreement between Abū ’l-Wafāʾ’s tangent table and our reconstruction is not only nearly perfect, it in fact even allows us to conclude that Abū ’l-Wafāʾ’s sine table was used for the calculation of his tangent table. To make this clear, first note that the nine errors in his sine table occur for arguments 19, 22, 25, 35, 44, 56, 67, 78 and 80°. If a sine table has an error for argument x, the tangent values computed from it will have errors for arguments x and $90° - x$. Within the range of arguments displayed in Table 3 the errors in Abū 'l-Wafāʾ's sine table would thus leave traces in the tangent values for arguments 65 (= 90 − 25), 67, 68 (= 90 − 22), 71 (= 90 − 19), 78, and 80°. Now the use of a correct sine table with values to sexagesimal thirds leads to tangent values for these arguments that differ by +5, +3, −7, −9, −6, and +6 thirds respectively from those of Abū 'l-Wafāʾ (these differences are not displayed in Table 3), whereas the use of Abū 'l-Wafāʾ's sine table leads to differences of 0, 0, −1, 0, −1, and 0 thirds (fourth column of Table 3). Thus there can be no doubt that Abū 'l-Wafāʾ's tangent table was in fact computed from his sine table.

Chapter 4 of [van Dalen 1993] contains various similar results that, in combination with perusal of *al-Majisṭī*, were decisive in the attribution of a total of nine tables from the *Baghdādī Zīj* to Abū 'l-Wafāʾ. For instance, a table for the equation of daylight for Baghdad with values to sexagesimal thirds [van Dalen 1993: 181–183] was shown to have been derived from another table by means of inverse linear interpolation in an accurate sine table with values to thirds for every 15 minutes of arc, rather than quadratic interpolation, values to seconds, or any other increment of the argument (the tables for the sine and tangent in *al-Majisṭī* in fact had values for every 15 minutes, so that strictly speaking the tables in the *Baghdādī Zīj* are extracts from those of Abū 'l-Wafāʾ).

Example 2. Table 4 shows the errors in a small part of the sine table from the *Sulṭānī Zīj* by Ulugh Beg (Samarkand, ca. 1440). This table can be regarded as the culmination of Islamic computational mathematics and displays values to five sexagesimal places (roughly 8 decimals) for every minute of arc from 0° to 87° and values to six sexagesimal places (10 decimals) between 87° and 90°. Most of the tabular values are correct; less than half of them contain errors of +1 or −1 (incidentally +2 or −2) in the final sexagesimal position. However, there are some peculiar groups of errors starting from argument 87°, precisely where the number of sexagesimal places is increased from 5 to 6. As can be seen from Table 4, the sine values for multiples of 5′ in this part of the table are correct (as in the whole table), whereas the errors in between these multiples are alternately positive and negative. A plausible explanation of these characteristics is the use of quadratic interpolation between accurately calculated values for every 5 minutes of arc (note that in the case of linear interpolation the errors in all groups would have had the same sign).

Generally, the use of interpolation in a trigonometric table can be recognized from the tabular differences: linear interpolation leads to groups of roughly constant first-order differences, quadratic interpolation to groups of roughly constant second-order differences. However, quadratic interpolation on intervals of 5′ produces such a good approximation to the sine that under normal circumstances it is practically impossible to distinguish between accurately computed and interpolated values. For the same reason, the second-order differences in Ulugh Beg's sine table change so slowly that separate groups cannot be recognized, except between 87° and approximately 87°25′, where the differences fluctuate around $-1'''5^{\text{iv}}44^{\text{v}}$ (see

arc	Ulugh Beg's sine	second-order differences	Ulugh Beg *minus* recomputation
86;50	59;54,30,11, 0	−1,5	
86;51	59;54,33,38,42	−1,6	
86;52	59;54,37, 5,19	−1,6	
86;53	59;54,40,30,50	−1,6	
86;54	59;54,43,55,15	−1,5	
86;55	59;54,47,18,34	−1,6	
86;56	59;54,50,40,48	−1,6	
86;57	59;54,54, 1,56	−1,5	
86;58	59;54,57,21,58	−1,5,46	−1
86;59	59;55, 0,40,55	−1,5,48	
87	59;55, 3,58,46,14	−1,5,48	
87;1	59;55, 7,15,31,40	−1,5,48	+11
87;2	59;55,10,31,11,18	−1,5,47	+16
87;3	59;55,13,45,45, 8	−1,5,48	+15
87;4	59;55,16,59,13,11	−1,5,40	+10
87;5	59;55,20,11,35,26	−1,5,38	
87;6	59;55,23,22,52, 1	−1,5,40	−8
87;7	59;55,26,33, 2,58	−1,5,40	−11
87;8	59;55,29,42, 8,15	−1,5,38	−11
87;9	59;55,32,50, 7,52	−1,5,46	−8
87;10	59;55,35,57, 1,51	−1,5,46	
87;11	59;55,39, 2,50, 4	−1,5,45	+5
87;12	59;55,42, 7,32,31	−1,5,46	+7
87;13	59;55,45,11, 9,13	−1,5,45	+7
87;14	59;55,48,13,40, 9	−1,5,42	+5
87;15	59;55,51,15, 5,20	−1,5,42	
87;16	59;55,54,15,24,49	−1,5,42	−3
87;17	59;55,57,14,38,36	−1,5,41	−5
87;18	59;56, 0,12,46,41	−1,5,42	−5
87;19	59;56, 3, 9,49, 5	−1,5,45	−3
87;20	59;56, 6, 5,45,47	−1,5,45	
87;21	59;56, 9, 0,36,44	−1,5,44	+3
87;22	59;56,11,54,21,56	−1,5,46	+3
87;23	59;56,14,47, 1,24	−1,5,44	+4
87;24	59;56,17,38,35, 6	−1,5,43	+2
87;25	59;56,20,29, 3, 4	−1,5,44	
87;26	59;56,23,18,25,19	−1,5,42	−1
87;27	59;56,26, 6,41,50	−1,5,44	−3
87;28	59;56,28,53,52,39	−1,5,43	−2
87;29	59;56,31,39,57,44	−1,5,45	−2
87;30	59;56,34,24,57, 6	−1,5,44	
87;31	59;56,37, 8,50,43	−1,5,45	+1
87;32	59;56,39,51,38,36	−1,5,45	+1
87;33	59;56,42,33,20,44	−1,5,44	+1
87;34	59;56,45,13,57, 7	−1,5,45	+1
87;35	59;56,47,53,27,46	−1,5,44	

Table 4. The use of second-order interpolation in the sine table of Ulugh Beg (the differences are given in thirds, fourths and, starting from 86°58′, fifths).

the third column of Table 4). So it is only thanks to the errors in this part of the table, apparently related to the change from 5 to 6 sexagesimal places, that we can recognize the use of second-order interpolation in the computation of Ulugh Beg's table of sines.

It can in fact be shown that a special type of interpolation, described in many Persian *zījes* from the 13th to 15th centuries and attributed to the tenth-century Iranian astronomer Abū Jaʿfar Muḥammad al-Khāzin [see Hamadanizadeh 1987], was used. With this type of interpolation, the tabular values for arguments between x and $x+n\cdot\Delta x$ (with n an integer) are calculated in such a way that they lie on the parabola through the points $(x-\Delta x,\ f_T(x-\Delta x))$, $(x,\ f(x))$ and $(x+n\cdot\Delta x,\ f(x+n\cdot\Delta x))$, where $f(x)$ and $f(x+n\cdot\Delta x)$ are accurately calculated tabular values for arguments x and $x+n\cdot\Delta x$ and $f_T(x-\Delta x)$ is the result of the application of the same type of interpolation to the preceding interval. Thus the errors in Ulugh Beg's sine values for arguments $87^\circ 1'$ and greater result from the less precise value of $\sin 86^\circ 59'$ that had to be used for their calculation (use of the value 59;55,0,40,55,15 would have avoided the following groups of errors).

As far as I know, Ulugh Beg's sine table is the first in which the use of this particular type of interpolation has been demonstrated. Unpublished research by the present author indicates that his set of tables of oblique ascensions for geographical latitudes 1° to 50° also relies heavily on this type of interpolation.

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