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COMMENTARY ON THE HOROSCOPE OF ISKANDAR SULTAN: AN INTRODUCTION TO ISLAMIC MATHEMATICAL ASTROLOGY

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1. Introduction

Iskandar Sultan (1384–1415) was the son of ʿUmar Shaykh, the eldest of the four sons of Tamerlane, and the Mongol princess Malikat Agha. Iskandar ruled over several parts of Iran after the early death of his father, was involved in the struggle for power over various parts of the Timurid empire after Tamerlane's death, and was finally executed on the order of his half-brother Rustam. Iskandar remains most well-known for his patronage of the arts and sciences and is associated with several beautifully illustrated manuscripts that survive from his time.

One of these manuscripts is his own deluxe Horoscope (i.e., nativity book), compiled in 1411 CE, which is extant as London, Wellcome Library, Persian 474.[1] Besides the famous, magnificent colour depiction of the horoscope for the moment of Iskandar's birth and predictions based on the astrological houses at the time of his birth and for the next twelve solar years of his life, this nativity book includes full details of all calculations needed to establish the horoscope as well as a large set of tables. In particular, the 86 folios that make up the manuscript contain the following:[2]

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[1] See https://wellcomecollection.org/works/ua87equq for a brief description of the manuscript and access to public-domain scans of the entire work. The manuscript was first introduced in Keshavarz, 'The Horoscope', and Elwell-Sutton, 'A Royal Tīmūrid Nativity Book', and was catalogued in Keshavarz, *A Descriptive and Analytical Catalogue*, pp. 396–401 (no. 224, with three colour plates). Further publications on the manuscript and its contents include Tourkin, 'Iskandar-sultan'; Tourkin, 'Medical Astrology', and Caiozzo, 'The Horoscope'. For a full analysis of a much later but similar nativity book, see Elwell-Sutton, *The Horoscope*.

[2] The manuscript was originally foliated in reverse order from 1 to 87 at the right top of right-side pages (with the beginning of the text of the Horoscope on fol. 87r, the illuminated horoscope diagram between fols 70 and 71, and the colophon on fol. 21r). Later on, the manuscript was re-foliated in the correct order at the top left of recto pages, but only on every tenth folio and some significant other pages; with this numbering, the text starts on fol. 1v, the horoscope diagram is on fols 18v–19r, and the colophon on

- Introduction (fols 1v–2r).
- Calculations of all quantities needed for establishing the horoscope (fols 2r–16r). These quantities include: true planetary positions for the times of conception and birth, astrologically relevant arcs on the celestial sphere for the planets and a set of fixed stars, the correction of the ascendant by three canonical methods, etc. Only in one case is a geometrical proof given, namely for the determination of the incidental horizon (see Section 19), but for most of the other calculations step-by-step results are provided. The text includes a defective diagram for the calculation of the astrological houses (fol. 4v, see p. 55 in Section 15), a problematic diagram of the orbs of Mercury filled in gold (fol. 5v, see p. 36 in Section 12), and an unclear diagram for the incidental horizon (fol. 6v, see p. 68). At the end of this chapter, Iskandar's life expectancy is estimated as 30 years, 10 months and 6 days (around four months shorter than his actual life-span).
- Tabular representation of the numerical results of the first section (fols 16v–20r, interrupted by the illustration of the birth horoscope). These include: a table of the ecliptic longitudes of the houses, planets, and around fifty astrological lots at the time of birth, with some astrological information; and a table with longitudes, latitudes, magnitudes, ascensions of transit, ascensions of rising, and temperaments at the time of birth for 63 fixed stars (fols 18r, 19v–20r).
- Double-page illustration of the birth horoscope of Iskandar Sultan (fols 18v–19r, with two pages of the star table on its obverse).
- General predictions for Iskandar based on the twelve astrological houses at the time of his birth, and a discussion of specific predictions based on prorogations (*tasyīr*s), termini (*intihā*ʾs), periods (*fardār*s) and ascendants of transfers (*ṭawāliʿ-i taḥwīlāt*) (fols 20v–23r).
- Tables of prorogations and termini for the twelve houses, the seven planets, and the lot of fortune for a period of 88 years starting with the year of Iskandar's birth (fols 23v–62v). This is supplemented by tables of termini of the ascendant for each month of the years in a twelve-year cycle (fols 63r–65r). Brief comments on both sets of tables can be found in Appendix E.

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fol. 68v. All my references (given both in the main text and in the margins) are to the later foliation in the correct order of the manuscript. Note that fol. 3 was incorrectly bound and in fact belongs after fol. 5; as a result, the topics in this commentary are discussed in the order fols 4, 5, 3. The scans of the manuscript on the website of the Wellcome Library were initially provided in disorder; the correct order was restored in late June 2022.

- Specific predictions for the twelve solar years of Iskandar's life following the compilation of the horoscope in 1411 CE (fols 65v–68v). This section ends with a colophon that gives the name of the author as Maḥmūd b. Yaḥyā b. Ḥasan b. Muḥammad al-Kāshī, known as ʿImād al-munajjim, and the date of completion of the work as 22 Dhū l-qaʿda 813 (18 March 1411), four years before Iskandar's death.
- Ephemeris for true solar (i.e., Jalālī) years corresponding to the Yazdigird years 781–791 (1412–1422) (fols 69r–84v). The table displays four months of the Persian calendar on every page (with five or six additional lines for the *epagomenae* in the second column of the third page for some, but not all of the years) and starts with the month Murdādh māh of the true solar year starting in 781 Yazdigird. The headers of the table indicate the Yazdigird equivalents of the Jalālī year beginnings (i.e., the vernal equinoxes) in the form 'Ephemeris (*taqwīm*) of the tenth year of the tenth [Saturn–Jupiter] conjunction in the airy triplicity, the night of Sunday, 10 Tīr māh [in the] old [calendar] of the year 782 [Yazdigird]'. The actual first page of the ephemeris, displaying the months Farwardīn to Tīr of the true solar year starting in 781 Yazdigird, may have been omitted; the heading of the current first page states 'corresponding to the night of 28 Dhū l-qaʿda of the year 784 [Hijra], hours elapsed since (*gudhashta az*) noon', suggesting that a first part similar to the headers for the following years is missing. For every month the following quantities are provided: the weekdays, 'Jalālī' (i.e., the day of the Persian month), and the true positions of the Sun, the Moon, the five planets and the ascending node of the Moon.
- Horoscope diagrams with brief predictions for the Yazdigird years 781–792 (1411–1422) (fols 85r–86r). The diagrams have headings similar to those for the years of the preceding ephemeris, e.g., 'Horoscope diagram (*zāʾirja*) of the ascendant of the ninth year of the tenth [Saturn–Jupiter] conjunction of the airy triplicity, corresponding to 781 Yazdigird' (this last part is omitted from the first diagram). That the diagrams are intended for the year transfers (i.e., the vernal equinoxes) of the respective years is confirmed by the fact that the position of the Sun is always indicated as 0;0° in the section for Aries. The central panel gives the termini (*intihāʾ*) of the conjunction, which increase by exactly one zodiacal sign per year, and the lords of the year (*sālkhudā*), mostly indicated by the last letter of the planets concerned. The sections for the twelve houses present the positions of important fixed stars besides those of the Sun, the Moon and the planets.

The first section of the Horoscope contains a large amount of mathematically computed astronomical and astrological data. The purpose of this commentary is to provide definitions for the astronomical and mathematical-astrological concepts that are used in this part of the Horoscope, and to outline the methods by which the data in the Horoscope were calculated. The commentary explains all computations in detail and verifies the correctness of the numbers given by the author. Furthermore, the accuracy of the calculations and, in some cases, the source of the errors in the data in the Horoscope are discussed. Since also basic concepts are explained, and most relevant aspects for the computation of horoscopes are treated, this commentary can also be read as an introduction to the mathematics of Islamic astrology.

A full English translation of the text of the nativity book of Iskandar Sultan was prepared by Sergei Tourkin in the early 2000s, but unfortunately remains unpublished. The present mathematical commentary on the Horoscope was originally written in 2003 and 2004 in close collaboration with Tourkin and has now been expanded with additional explanations and relevant recent literature. Since it is not possible to refer to the translation, references to folio and line numbers in the manuscript are provided in marginal notes (and occasionally in the main text) for all discussions of topics and results of calculations that are provided in the Horoscope.

2. Sexagesimal numbers

Nearly all numbers in the Horoscope, and in Islamic astronomical and astrological sources in general, are written in the sexagesimal number system that had been common in astronomical practice from ancient Babylonian times onwards. In Arabic and Persian, the *abjad* system, an adaptation of the Greek notation for sexagesimal numbers, was adopted. In this system, single letters of the Arabic alphabet represent the numbers from 1 to 9, the multiples of 10 up to 90, and the multiples of 100 up to 1000: *alif* = 1, *bāʾ* = 2, *jīm* = 3, ..., *ṭāʾ* = 9, *yāʾ* = 10, *kāf* = 20, *lām* = 30, ..., *sīn* = 60, *qāf* = 100, *rāʾ* = 200, ..., *ghayn* = 1000. These letters were combined to denote numbers such as 25 = *kāf-hāʾ* كه, 47 = *mīm-zāʾ* مز, 358 = *shīn-nūn-ḥāʾ* شنح, 1962 = *ghayn-ẓāʾ-sīn-bāʾ* غظسب, etc. The following table shows the letters of the Arabic alphabet with their values as *abjad* numbers, as well as the special symbol for zero, which derives from the Greek form ō, an omicron with a macron.[3]

[3] A slightly different system became common in the western Islamic world; it uses, for example, *ṣād* instead of *sīn* for 60. For the origin of this system, see Thomann,

0	٦				
1	ا	10	ى	100	ق
2	ب	20	ک	200	ر
3	ج	30	ل	300	ش
4	د	40	م	400	ت
5	ه	50	ن	500	ث
6	و	60	س	600	خ
7	ز	70	ع	700	ذ
8	ح	80	ف	800	ض
9	ط	90	ص	900	ظ
				1000	غ

In publications on Islamic astronomy, sexagesimal numbers are usually transcribed according to the following convention, introduced by Otto Neugebauer: sexagesimal digits are separated by commas, and the sexagesimal point is represented by a semicolon. Thus 3,12;38,45 denotes $3 \cdot 60 + 12 + \frac{38}{60} + \frac{45}{60^2}$. Following the Greek usage, in Arabic and Persian the integer part of a sexagesimal number larger than 60 is mostly expressed decimally, so that the above number may be more frequently encountered as 192;38,45.[4] In general, only planetary longitudes, centrums and anomalies are expressed in zodiacal signs plus degrees. Only in the case of ecliptic longitudes do these signs correspond to the actual signs of the zodiac, in all other cases they are simply a measure of thirty degrees. In transliterations of sexagesimal numbers, zodiacal signs are denoted by a superscript ‘s’. Here 0^s stands for the first sign, Aries, i.e., ecliptic degrees between 0 and 30; 1^s stands for Taurus, 2^s for Gemini, etc. Consequently, the longitude of the ascendant at the time of birth of Iskandar is written as $9^s\,0;17,35^\circ$, equivalent to $0;17,35^\circ$ Capricorn (cf. Section 14). In general, arcs and angles are followed by a degree sign, while plain numbers are not. Since the Persian text does not explicitly indicate zodiacal signs or the place of the sexagesimal point,

‘Scientific and Archaic Arabic Numerals’. For the forms of *abjad* numerals as they are found in the manuscripts, see Irani, ‘Arabic Numeral Forms’.

[4] If, however, in such cases the integer part is larger than 360 (i.e., the number of degrees in a circle), it is often written with the so-called Hindu numerals, the precursors of the Arabic and western digits from 0 to 9. For these see, for example, the article ‘ʿIlm al-ḥisāb’ by Abdelhamid I. Sabra in *The Encyclopaedia of Islam. New edition*, vol. III (1971), pp. 1138–1141; Lemay, ‘The Hispanic Origin’; Kunitzsch, ‘The Transmission’; and Burnett, ‘Indian Numerals’. For the forms of Hindu numerals as they are found in medieval Arabic and Persian manuscripts, see Irani, ‘Arabic Numeral Forms’.

the number that is intended must often be determined from the context. For example, 2 21 18 43 could not only stand for $2^s21;18,43°$, but also for 2,21;18,43°, 2;21,18,43°, 0;2,21,18,43, etc. The similarity of certain Arabic and Persian letters leads to another difficulty in the interpretation of the numbers in the text, namely the possibility of scribal mistakes. Pairs of letters such as ج and ح (3 and 8), د and ز (4 and 7), و and ز (6 and 7), ط and ك (9 and 20), and ى and ن (10 and 50, in compounds such as يا 11 and نا 51) can be easily confused, especially because the dots were often omitted.[5]

Sexagesimal calculations may be carried out by means of my computer program SCTR, a sexagesimal calculator originally written for DOS, but now also available in a 32bit version for Microsoft Windows 7 and later (download from my website http://www.bennovandalen.de/). For carrying out the calculations in this article, I also made use of my program Historical Horoscopes for determining planetary mean positions and equations and calculating an horoscope for a given moment in time on the basis of a set of parameters from a historical source. Furthermore, I made use of ad hoc programs for calculating quantities such as the distance from the equator, ascension of transit, rising and setting, latitude of the incidental horizon, and projection of the rays.

An important objective for writing this commentary was to account for the interpretation of the sexagesimal numbers in the text of Iskandar's Horoscope, to correct the scribal mistakes in the manuscript, and to explain certain errors in the calculations. In order to achieve this, I checked nearly every single computation in the text.[6] Thus I came to the conclusion that the calculations were generally carried out systematically and accurately. Of the existing errors, most were found to be due to reading mistakes or computational errors by the author of the Horoscope himself, since these errors propagate through the following steps of the calculations, leading to an erroneous final result. This seems to indicate that the results of the calculations were generally entered into the Horoscope directly without checking and correcting. Only relatively few errors can be identified as plain scribal ones.

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[5] For a more extensive discussion of possible scribal errors in *abjad* numbers, see van Dalen, 'The Geographical Table', pp. 532–534, and van Dalen, *Ptolemaic Tradition*, p. 75, note 1 and the further references given there.

[6] For all recomputations I have assumed the use of the standard type of rounding (i.e., digits 30 and higher are rounded upwards, digits 29 and lower downwards) that was mostly used by Ptolemy and Islamic astronomers, rather than truncation. Errors in the values in the Horoscope of 10 seconds or less are not always separately mentioned in the 'notes on the calculations'.

3. Chronology

A variety of essentially different calendars were described and used in Islamic astronomical sources. This concerned, on the one hand, the calendars in use by the various people that had become part of the Islamic realm, and on the other hand historical calendars in which the dates were expressed of observations that were used to improve the accuracy of astronomical parameters, especially of planetary mean motions. The calendars referred to included:

- Solar calendars with a constant year length of exactly 365 days, namely the ancient Egyptian calendar and the most common form of the Persian calendar named after the last king of the Sassanid dynasty, Yazdigird III (r. 632–651).
- Solar calendars with an ordinary year length of 365 days and a leap year of 366 days every fourth year, namely ‘Roman’ and ‘Syrian’ versions of the Julian calendar (with the Seleucid era in 311 BCE as the most common epoch) as well as the Coptic calendar with its year beginning on 30 August and twelve months of 30 days plus 5 or 6 so-called *epagomenal* days (usually reckoned from the era of Augustus in 30 BCE or the era of Diocletian in 284 CE).
- True solar calendars in which the first day of the year is determined from the exact time of the vernal equinox, in particular the Malikī or Jalālī calendar introduced by the Seljuk sultan Jalāl al-Dawla Malikshāh I (r. 1072–1092) with epoch 1079. This calendar used the month names of the traditional Persian calendar (distinguished by an indication ‘Jalālī’ as opposed to *qadīm* ‘old’) and made the *epogomenae* either five or six days.
- The purely lunar Arabic calendar used by Muslims for both religious and civil purposes. The epoch of this calendar is the *hijra* (the flight of the prophet Muḥammad from Mecca to Medina in 622 CE) and its years are approximately eleven days shorter than a solar year.
- Occasionally also descriptions of lunisolar calendars (with each year consisting of either twelve or thirteen lunar months in such a way that the average year length becomes equal to that of the true solar year) may be found in Islamic sources. This includes, in particular, the Jewish calendar and the Chinese calendar that was introduced to Persia by the Mongol conquerors in the 13th century.

Of these calendars, the three most important ones were the Arabic, Persian and Julian calendars. The characteristics of these calendars and rules for converting dates between them were described in most Islamic astronomical handbooks.[7]

2r: 21–2v: 3 The birth of Iskandar Sultan is said in the Horoscope to have taken place in the night to Monday, 3 Rabīʿ al-awwal 786 Hijra (25 April 1384). This date is in full agreement with the Maliki, Yazdigird and Seleucid dates given in the following lines of the text, namely 15 Urdibihisht 306 Malikshāh, 17 Murdādh 753 Yazdigird, and 25 Nīsān 1695
2v: 9–16 Alexander. A few sections later the time of the nativity is said to have been observed as 4;0 equal hours after sunset. The time of sunset on the day in question can be determined from data that are given in the following lines (lines 12–14; these calculations will be explained in more detail in Sections 7, 16 and 18): the true solar longitude is found as 1^s 12;38,45°, and the solar declination as 15;40,18° north. Using the latitude for Uzgand, given as 44;0° on fol. 2v: 7, we find the equation of daylight as 15;43,10°, corresponding to approximately one hour and three minutes. Thus, on the day of the nativity, the sun set at 7:03 pm mean local time. It follows that Iskandar was born just after 11 pm, which is in reasonably good agreement with the time expressed in terms of the Chinese-Uighur calendar (cf. below). Since for Muslims the day starts at sunset, for them the nativity took place on Monday, but expressed in the Julian calendar it was on Sunday, 24 April 1384, one hour before midnight. The formulation of the section on the 'Time of the Blessed Nativity' (fol. 2v: 9–10) suggests that 11 pm was the *true* local time of the nativity, i.e., the time determined directly from solar observations. This will be confirmed in Section 7.2 and Appendix C, where I explain that this true time is (implicitly) converted to *mean* time (on which the planetary tables in astronomical handbooks are based) in order to calculate the other planetary positions. I will show that the determined mean time of the nativity must have been close to 10;40,13 hours after noon (see p. 25).

2r: 25–2v: 3 The data in the Horoscope concerning the Chinese-Uighur calendar have already been examined in detail by Elwell-Sutton.[8] In Appendix A,

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[7] For an overview of the calendars and the methods for converting dates, see Ginzel, *Handbuch*; Taqizadeh, 'Various Eras and Calendars' (2 parts), and the article 'Taʾrīkh' by F. C. de Blois and Benno van Dalen in *The Encyclopaedia of Islam. New edition*, vol. X (2000), pp. 258–271, which was partially reprinted in a new format in van Dalen, 'Dates and Eras'. Checks of the equivalence of dates in calendars used in the Islamic world are traditionally carried out by means of Spuler & Mayr, *Wüstenfeld-Mahler'sche Vergleichungs-Tabellen*. More convenient and comprehensive checks can be performed by means of my own DOS-like program CALH, the Windows program Kairos by Raymond Mercier, and other tools available on the internet.

[8] See Elwell-Sutton, 'A Royal Tīmūrid Nativity Book', pp. 121–123.

I supplement his conclusions with some more information on the basis of a later study of the technical foundations of the calendar as they were expounded in the *Īlkhānī Zīj* by Naṣīr al-Dīn al-Ṭūsī.[9] Both the day and the time indicated for the Chinese-Uighur calendar turn out to be in perfect agreement with the other chronological data found in the Horoscope.

4. Geography

Many calculations in spherical astronomy depend on a value for the geographical latitude of the locality for which they are performed. Calculations of solar, lunar and planetary positions depend on local time and therewith on the geographical longitude of the locality. The direction of prayer in Islam, the *qibla*, depends on the geographical coordinates of Mecca and on those of the locality at which the prayer is performed. For these reasons, tables with geographical coordinates were essential aids for many types of astronomical and astrological calculations. Such tables were found in numerous Islamic astronomical handbooks[10] and in various other types of works, as well as on certain astrolabes.[11]

Longitudes in Islamic sources were mostly measured with respect to a zero meridian to the west of the known part of the inhabited world. This included the meridian of the 'Fortunate Isles' (i.e., the Canaries, Ptolemy's traditional meridian of reference), the 'shore of the sea' (i.e., the westernmost coast of Africa, used in the huge new survey of geographical coordinates produced by the scholars working for the early Abbasid caliph al-Maʾmūn around 830 and taken to lie 10 degrees east of the meridian of the Fortunate Isles), and in later centuries also the so-

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[9] See van Dalen et al., 'The Chinese-Uighur Calendar'. For an edition and translation of the text and tables in the *Īlkhānī Zīj*, see Isahaya, 'The *Tārīkh-i Qitā*'. For the origin of the Chinese calendar in Islamic sources, see Isahaya, 'History and Provenance'.

[10] Islamic astronomical handbooks with tables (in Arabic / Persian called *zīj*, pl. *zījāt* or *azyāj*) were the most important tool for any practising astronomer or astrologer. Their central part was a large set of tables for calculating planetary longitudes and latitudes at any given time. Further tables for trigonometrical, spherical-astronomical and specifically astrological functions also allowed the compilation of basic horoscopes or a full-blown nativity book such as the one discussed here. For overviews of the most important zījes and their contents, see Kennedy, 'A Survey' and King & Samsó, 'Astronomical Handbooks and Tables'. A new, more extensive survey is being prepared by the present author.

[11] See Kennedy & Kennedy, *Geographical Coordinates* for ordered lists of the geographical coordinates in more than 80 Islamic sources. An extended and corrected version of this work was prepared by the late Mercè Comes and unfortunately remains unpublished. For a recent analysis of a geographical table making extensive use of the data collected by the Kennedys, see van Dalen, 'The Geographical Table'.

called ‘meridian of water’ (used by scholars in the western-Islamic world as a result of a correction of earlier values for the length of the Mediterranean).[12] If planetary positions are calculated for a certain locality by means of astronomical tables intended for another city, a correction for the difference in geographical longitude between the two places needs to be applied. For example, if al-Battānī’s tables for Raqqa (longitude 73;15° from the Fortunate Isles) are used to calculate a planetary longitude at Baghdad (longitude 80°), a correction for a longitude difference of 6;45° in eastern direction must be carried out. Since a longitude difference of 15° corresponds to 1 hour, local noon at Baghdad precedes noon in Raqqa by 27 minutes. Therefore the planetary position at noon in Baghdad is found as that for $11^{h}33^{m}$ at Raqqa.

2v: 4–8 According to his Horoscope, Iskandar Sultan was born at Uzgand, now Uzgen in Kyrgyzstan. The geographical coordinates of this locality are given in the text as 102;50° east of the Fortunate Isles and 44;0° north. These are the coordinates attributed to the city in the geographical tables of such astronomical handbooks as the *Īlkhānī Zīj* by Naṣīr al-Dīn al-Ṭūsī (Maragha, *c.* 1270), the *Khāqānī Zīj* by Ghiyāth al-Dīn Jamshīd al-Kāshī (Kashan, *c.* 1410), and the *Zīj* of Ulugh Beg (Samarkand, *c.* 1440).[13] However, the longitude was essentially already found with al-Bīrūnī (active in Khwarazm and Afghanistan, *c.* 1000), who placed Uzgand 92;50° east of the western shore of Africa, corresponding to 102;50° east of the Fortunate Isles.[14] In Section 7 below I will show that the *Īlkhānī Zīj* was used for all calculations of planetary longitudes in the Horoscope of Iskandar Sultan. Since the base locality of the *Īlkhānī Zīj* is Maragha in northwestern Iran, a correction for the difference in geographical longitude between that city and Uzgand needs to be applied in all planetary calculations. Maragha is given a longitude of 82;0° with respect to the Fortunate Isles in the *Īlkhānī Zīj* and most later sources (whereas al-Bīrūnī had 73;20° with respect to the western

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[12] See Kennedy & Kennedy, *Geographical Coordinates*, p. xi, and Comes, ‘The “Meridian of Water”’.

[13] cf. Kennedy & Kennedy, *Geographical Coordinates*, pp. 510, 559, and 564. The coordinates in the zījes of al-Ṭūsī, al-Kāshī and Ulugh Beg mostly stem from the mysterious *Kitāb al-Aṭwāl wa-l-ʿurūd li-l-Furs*, which is itself lost but whose coordinates for 452 localities were quoted by the 14th-century prince and geographer Abū l-Fiḍāʾ. This work has tentatively been dated to the 13th century, although a large subset of its coordinates are already found in the early 12th-century *Dustūr al-munajjimīn*. See Kennedy & Kennedy, *Geographical Coordinates*, pp. xvii and 422–430; Sezgin, *Geschichte des Arabischen Schrifttums*, vol. XIII, pp. 369–375, and van Dalen, ‘The Geographical Table’, p. 544.

[14] Kennedy & Kennedy, *Geographical Coordinates*, p. 459.

shore of Africa), so that the correction amounts to 20;50°. As we will see, the planetary mean motions are in very good agreement with this correction. However, the actual difference in geographical longitude between Uzgand and Maragha amounts to as much as 27° (73;14° east of Greenwich vs 46;13°).

5. Trigonometric functions

In Islamic mathematics, the trigonometric functions sine and tangent were usually expressed in a base circle with a radius of 60 units (rather than the modern value 1 or various Indian values such as 150 and 3438 minutes).[15] The resulting functions are conventionally denoted as Sin and Tan, i.e., $\mathrm{Sin}\,x = 60 \cdot \sin x$ and $\mathrm{Tan}\,x = 60 \cdot \tan x$, for example $\mathrm{Sin}\,60° = 51;57,41 \approx 51.9615$ and $\mathrm{Tan}\,60° = 103;55,23 \approx 103.9230$. As was explained in Section 2, in the original Persian text of the Horoscope, as well as in Islamic astronomical and astrological sources in general, these two numbers would occur as 51 57 41 and 103 55 23 without indication of the position of the sexagesimal point. As a result it may be difficult in some cases to distinguish whether a base circle of 60 or 1 was used. In the Horoscope of Iskandar Sultan there are only very few places where we can definitely say which radius of the base circle was used for the sines and tangents. These include a few calculations in which an intermediate result is 'lowered' (*munḥaṭṭ*), i.e., its sexagesimal point is shifted to the left by one sexagesimal position, which is equivalent to dividing by 60.[16] Furthermore, one (non-existing) sine and one tangent are expressed with a first sexagesimal digit larger than 60. Some references occur to the 'maximum sine' (*jayb-i aʿẓam*), but this could stand for 1 as well as for 60. On the other hand, there are many calculations where a necessary division by the radius of the base circle (i.e., 'lowering') is left out, typically in calculations such as $\delta(\lambda) = \mathrm{arcSin}(\mathrm{Sin}\,\varepsilon \cdot \mathrm{Sin}\,\lambda/60)$, where $\delta(\lambda)$ stands for the declination of a point on the ecliptic with longitude λ, and ε is the obliquity of the ecliptic (cf. Section 16). I will nev-

[15] Ptolemy did not yet use the sine, but his table for chords in the *Almagest* likewise uses a radius of 60 units. Islamic astronomers frequently tabulated the cotangent for base 7 or 12, thus giving the shadow of a traditional gnomon of length 7 feet or 12 fingers as a function of the solar altitude. For a comprehensive history of trigonometry up to the sixteenth century, see Van Brummelen, *The Mathematics of the Heavens*.

[16] Calculations in which an intermediate result is explicitly 'lowered' are those of the equation of daylight for the Moon and the five planets on fols 6r–v (Section 18, but cf. p. 78 for the equation of daylight of fixed stars); an intermediate step in the explanation of the calculation of the incidental horizon on fol. 6v: 27, and the same step in the actual calculations for the Moon and Saturn on fol. 7r: 21, 27 (Section 19).

ertheless assume that all trigonometric functions in the Horoscope were expressed to base 60, and that in the above-mentioned cases the necessary divisions by 60 were simply omitted from the text. Thus I will refer to the sine of 60° as 51;57,41. However, in formulas I will write 'sin' and 'tan' throughout for the sake of clarity and will leave out the divisions by 60 even in cases where they occur explicitly in the text. Thus I will present the formula for the solar declination as $\delta(\lambda) = \arcsin(\sin\varepsilon\cdot\sin\lambda)$ rather than the one given above. I will also write '$\cos x$' as an abbreviation for the Persian *jayb-i tamām-i x* ('sine of the complement of x').

6. Spherical Astronomy

Besides the computation of planetary longitudes and latitudes (for which see Sections 7 and 8), most of the calculations in the Horoscope of Iskandar are of angles or sides of spherical triangles. Ancient Greek and medieval Islamic astronomers made use of various relations between the angles and sides of spherical triangles that have been taught until recent times, for instance, relations equivalent to the sine and cosine rules. Ptolemy also used a more general tool, the Theorem of Menelaus, that allowed the derivation of relations between the sides of several triangles bounded by three or four great circles. In the later Islamic period the so-called 'Rule of Four' became very popular. Below I present the rules for spherical triangles that are used in the Horoscope and in our commentary to calculate various quantities of astrological importance.[17]

I will start by giving some basic definitions. Planets and stars are seen as if they move on a sphere around the centre of the Earth, called 'the celestial sphere'. A *great circle* on a sphere is the intersection with that sphere of any plane through its centre; the centre of a great circle thus coincides with the centre of the sphere. As shown in Figure 1, examples of great circles on the celestial sphere are the celestial equator, which is perpendicular to the axis of revolution of the Earth and the

[17] For a more extensive modern treatment the reader is referred to textbooks on spherical astronomy such as Smart, *Textbook*. Van Brummelen, *The Mathematics of the Heavens* also discusses many of the spherical-astronomical theorems used in Islamic astronomy. Debarnot, *Kitāb Maqālīd* contains a study of early Islamic spherical astronomy and an edition and French translation of an important treatise by al-Bīrūnī (*c.* 1000). Kennedy, 'Spherical Astronomy' investigates the spherical astronomy of Ghiyāth al-Dīn Jamshīd al-Kāshī (d. 1429), who finished his *Khāqānī Zīj* only a few years after the Horoscope for Iskandar was completed and who worked for Iskandar in the same period as the author of the Horoscope (Keshavarz, 'The Horoscope', p. 198 suggests that the author of the Horoscope was the grandfather of the famous mathematician and astronomer).

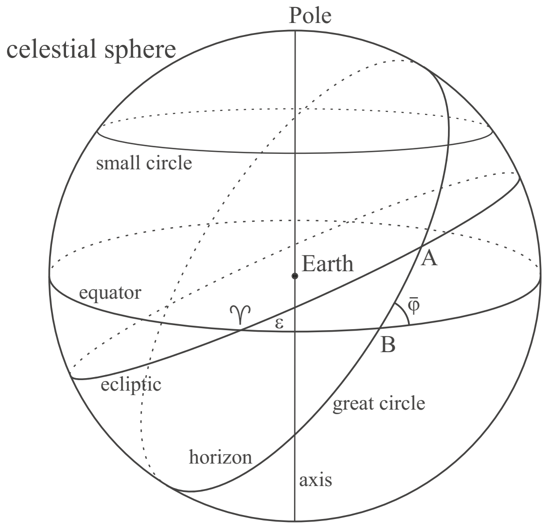

Figure 1: The celestial sphere as seen from the outside

celestial sphere and hence lies in the same plane as the Earth's equator; the ecliptic, the annual apparent path of the Sun around the Earth; and the horizon, which is parallel to the plane tangential to the Earth at the position of the observer. A *small circle* is any circle on the sphere that is not a great circle (and hence whose centre does not coincide with that of the sphere). An example of a small circle on the celestial sphere is the daily path of a star with non-zero declination (i.e., not located on the equator), which lies parallel to the equator. The poles of a circle on the sphere are the intersections with the sphere of the perpendicular through the circle's centre; thus, in Figure 1, *P* is a pole of the equator and of the unspecified small circle. Elementary theorems on great circles that will be used here include: 'two great circles intersect in two opposite points of the sphere (i.e., separated by arcs of 180° on either great circle)'.

A *spherical angle* is an angle included by two great circles on the sphere at one of their points of intersection. It is hence equal to the angle between the planes in which the two great circles lie. In Figure 1, ε is the spherical angle between the equator and the ecliptic; it is called the obliquity of the ecliptic and in Islamic sources has a size of between 23;30°and 23;35° (Ptolemy's value was 23;51,20° and the current value is 23;26,11°). The spherical angle $\bar{\varphi}$ between the horizon and the equator is the complement of the geographical latitude φ of the observer at the centre of the celestial sphere.

A *spherical triangle* consists of the arcs of three great circles between their points of intersection. The sides of a spherical triangle are measured by the angles that they subtend at the centre of the sphere. Note that thus both angles and sides of a spherical triangle are measured in degrees (or radians). In Figure 1, ♈AB is the spherical triangle enclosed by the equator, the ecliptic and the horizon; its sides are the angles $\angle ♈EA$, $\angle ♈EB$ and $\angle AEB$, where E is the centre of the Earth and of the celestial sphere.

Now let a, b and c be the sides of a spherical triangle and A, B and C the angles opposite a, b and c, respectively. If the triangle has one or more right angles, we assume $C = 90°$. When two triangles are compared, the arcs of the second triangle are denoted by a', b' and c' and its angles by A', B' and C'. The following rules can be shown to hold:[18]

Sine rule. In every spherical triangle, we have $\dfrac{\sin a}{\sin A} = \dfrac{\sin b}{\sin B} = \dfrac{\sin c}{\sin C}$.

Cosine rule. In every spherical triangle, we have $\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$, and analogous expressions for $\cos b$ and $\cos c$. In the case of a right-angled triangle ($C = 90°$), the expression for $\cos c$ generalizes to $\cos c = \cos a \cdot \cos b$, the Pythagorean formula for spherical astronomical triangles.

Rule of Four. If a pair of right-angled spherical triangles ABC and $A'B'C'$ has an acute angle $A = A'$ in common, then

$$\frac{\sin a}{\sin c} = \frac{\sin a'}{\sin c'} \qquad \text{and} \qquad \frac{\tan a}{\sin b} = \frac{\tan a'}{\sin b'}$$

'Double-tangent rule'. In a right-angled triangle, we have $\tan A = \tan a / \sin b$. This can be proved by means of the Rule of Four: Let triangle $A'B'C'$ be a right-angled triangle with angle A' equal to A and side b' equal to $90°$. The desired rule now follows at once from the tangent version of the Rule of Four, because $\sin b' = 1$ and $\tan a' = \tan A' = \tan A$.

7. Planetary Longitudes

The computation of the positions of heavenly bodies lies at the foundation of every practical astronomical and astrological activity. The Sun,

[18] Note that no actual mathematical formulas are found in the Horoscope or any other medieval mathematical or astronomical work. The procedures for calculating the various quantities discussed in the Horoscope are all written out in words in the primary sources. As indicated in Section 5, multiplications and divisions by 60, which are necessary when calculating with the standard radius of the base circle in Greek and Islamic trigonometry, are here omitted.

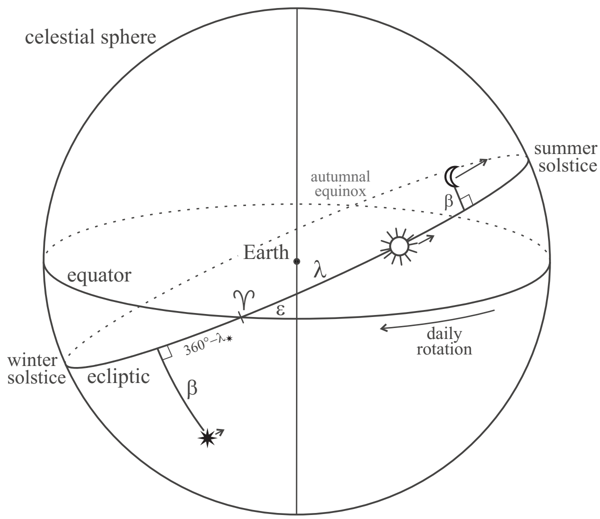

Figure 2: The celestial sphere with ecliptic coordinates

the Moon, the planets visible to the naked eye, and the fixed stars are seen by an observer on the Earth as if they move on a sphere around the centre of the Earth, the celestial sphere (cf. Section 6 and see Figure 2). In this model the daily rotation of the Earth around its axis is represented by the daily rotation (in western direction) of the entire celestial sphere around the axis of the equator. The annual rotation of the Earth around the Sun is observed as an annual rotation of the Sun (in eastern direction) around its path on the celestial sphere, the ecliptic. The point where the Sun crosses the plane of the equator from south to north is the vernal equinox (indicated by the symbol ♈), since it marks the time when day and night have equal lengths at the start of spring. The point opposite it, where the Sun crosses the plane of the equator from north to south, is the autumnal equinox. The point where the Sun reaches its largest northern distance from the equator is the summer solstice, the point opposite it the winter solstice. The Moon (in a period of little less than a month) and the five planets (in periods of one up to thirty years) also appear to rotate around the Earth in eastern direction, similar to the Sun. However, their positions periodically deviate from the ecliptic by up to around 8 degrees in northern or southern direction. Furthermore, the five planets occasionally change the direction of their motion to become retrograde. The fixed stars appear fixed with respect to each other (their ‘proper motion’ only became known in modern times), but their positions with respect to the ecliptic change very slowly in the course of time due to the precession of the equinoxes.

The positions of the heavenly bodies are generally measured with respect to the ecliptic (although the positions of fixed stars are also occasionally given in equatorial coordinates). The *longitude* of a heavenly body is the distance between the vernal equinoctial point and the orthogonal projection of the body onto the ecliptic, measured in degrees along the ecliptic and in eastern direction (in Figure 2, λ indicates the solar longitude). The *latitude* of a heavenly body is its distance from the ecliptic, measured in degrees and orthogonally to the ecliptic (in Figure 2, β indicates the latitudes of the Moon and of a fixed star). Starting with the vernal equinox, the ecliptic is divided into the twelve signs of the zodiac, which each have a length of 30°. Thus an ecliptic longitude can be expressed in degrees from 0 to 359 as well as in degrees from 0 to 29 within the zodiacal signs. For example, the true solar position at the preliminary time of Iskandar's birth (fol. 2v: 14) can be written as 42;38,45°, 1^{s} 12;38,45°, or 12;38,45° Taurus. As was explained in Section 2, in the notation 1^{s} 12;38,45°, the signs stand for multiples of 30°. This implies that a longitude in the first zodiacal sign, Aries, is indicated by 0^{s} and a longitude in the twelfth zodiacal sign, Pisces, by 11^{s}.

For the calculations of horoscopes, the positions of the heavenly bodies at the times of birth and conception were of particular importance. In the Horoscope of Iskandar Sultan we find not only the longitudes themselves, but also details of their calculation. In particular, step-by-step determinations of the following longitudes are included:

2v: 12–14 1. Solar longitude at the time of the nativity (fol. 2v, for determining the ascendant).

2v: 25–27 2. Longitude of Venus at the time of the nativity (fol. 2v, in applying Ptolemy's method for correcting the ascendant; see the computational example on pp. 21–22).

4r: 9–14 3. Lunar longitude at the time of the nativity (fol. 4r, in applying Hermes's method for correcting the ascendant).

4r: 17–19 4. Solar longitude at midnight of the day of conception (*idem*).

4r: 22–26 5. Lunar longitude at the exact time of conception (*idem*).

5r: 1–2 6. Planetary longitudes at the time of the nativity (fol. 5r; because the preceding folio is missing, only the last part of the calculation for Mercury has survived).

16r: 1–22 7. Planetary longitudes at the time of conception (fol. 16r).

Judging from the intermediate computational steps included in the Horoscope, we may assume that all planetary positions in the work were determined on the basis of the geocentric, geometrical planetary models as originally developed by Ptolemy in his *Almagest*. These models have

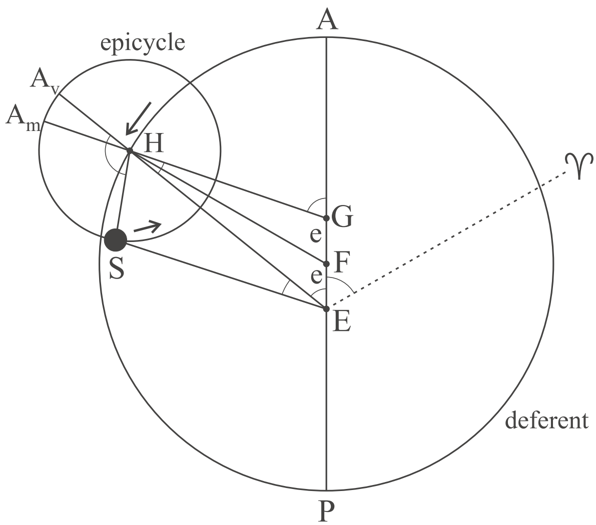

Figure 3: Ptolemy's planetary model

been explained in detail in numerous publications.[19] Here I will concentrate on those properties of the models that are needed to understand the consecutive steps in the calculations in the Horoscope and for determining the characteristics of the tables from which the planetary data were computed.

7.1. Ptolemy's Planetary Models

The model for the planets Saturn, Jupiter, Mars, and Venus is depicted in Figure 3. The planet *S* moves with a uniform angular velocity and counter-clockwise on a small circle, called the *epicycle*,[20] with centre *H*. The centre of the epicycle, in its turn, moves counter-clockwise on a larger circle, called the *deferent* ('carrier'), with centre *F*. Point *F* is a distance *e*, the *eccentricity*, removed from the Earth *E*. The motion of the centre of the epicycle is uniform with respect to a third point *G*, a distance 2*e* removed from the Earth in the same direction as *F*. (The circle around *G* with the same unit radius as the deferent, the *equant*, has been omitted from the figure for clarity). The apogee *A* of the deferent

[19] See, for instance, Neugebauer, *HAMA*, Book I, Sections B–C; Pedersen, *A Survey*, Chapters 5, 6, 9 and 10; and Van Brummelen, *Mathematical Tables*, Chapters 5, 8, 9 and 12. An English translation of the *Almagest* can be found in Toomer, *Ptolemy's Almagest*, in which the solar, lunar and planetary theories are developed in Books III–V and IX–XII.

[20] The Arabic/Persian terminology related to the models for planetary longitude is listed in Table 1 on p. 18 together with literal and common English translations.

English	*literal translation*	*Arabic/Persian*
deferent	deferent, 'apogee orb'	*ḥāmil*, *falak-i awjī*
dirigent	'director'	*mudīr*
epicycle	epicycle, epicyclic orb	*[falak-i] tadwīr*
eccentricity*	'between the two centres'	*mā bayn al-markazayn*
equant	'equaliser of motion'	*muʿaddil al-masīr*
apogee (deferent)	'farthest distance'	*buʿd-i abʿad*, *awj*
perigee (deferent)	perigee	*ḥaḍīḍ*
mean centrum	'centrum'	*markaz*
true centrum	'adjusted centrum'	*markaz-i muʿaddal*
equation of centre	'[first] equation'	*taʿdīl[-i awwal]*
mean apogee (epicycle)	apogee	*dhirwa*
true apogee (epicycle)	true apogee	*dhirwa-yi ḥaqīqī*
mean anomaly	'anomaly'	*khāṣṣa*
true anomaly	'adjusted anomaly'	*khāṣṣa-yi muʿaddala*
equation of anomaly (at average distance)	'second equation'	*taʿdīl-i thānī*
differences in the equation of anomaly	'difference'	*ikhtilāf*
interpolation minutes	'minutes of the arguments'	*daqāʾiq-i ḥiṣaṣ*
equation of anomaly	'adjusted equation'	*taʿdīl-i muʿaddal*
corrected centrum	'corrected centrum'	*markaz-i muqawwam*
longitude, true longitude	'true position'	*taqwīm[i- muʿaddal]*
equation of time	'equation of days and their nights'	*taʿdīl al-ayyām bi-layālīhā*
being in direct motion		*mustaqīm*
being in retrograde motion		*rājiʿ*

Table 1: Terminology of the models for planetary longitude (an asterisk indicates terminology that does not actually occur in the Horoscope)

is the point furthest removed from the Earth, the perigee P is the point nearest to the Earth. Note that A, G, F, E and P all lie on a straight line.

In order to calculate the longitude of the planet, we first find the *mean centrum* (in the Horoscope simply called 'centrum'), i.e., the angle $\angle AGH$ (measured from the apogee) under which the epicycle centre would be seen from point G. The mean centrum is a linear function of time, which can be found from the tables of mean motion in any zīj. It is converted to the *true centrum* (in the text: 'adjusted centrum'), the angle $\angle AEH$ (likewise measured from the apogee) under which the epicycle centre is seen from the Earth, by adding or subtracting $\angle EHG$, the *equation of centre* (in the text: 'equation' or 'first equation', to be understood as 'correction').[21] Note that the equation of centre must be subtracted

[21] The planetary equations can be calculated by applying theorems of plane trigonometry, in particular, the Theorem of Pythagoras, to right-angled triangles obtained by extending sides of some of the triangles seen in Figure 3; for full details, see the references given in footnote 19.

from the mean centrum if the epicycle centre is located in between the apogee and the perigee of the deferent, i.e., if the mean centrum lies between 0° and 180°, and added otherwise.

We now consider the motion of the planet S on the epicycle. The *mean apogee* A_m of the epicycle is the intersection with the epicycle of the extension of the line GH beyond H. Similarly, the *true apogee* A_v is the intersection with the epicycle of the extension of the line EH beyond H. The mean apogee is the point of the epicycle relative to which the planet moves with a uniform angular velocity, while the true apogee is the point of the epicycle furthest removed from the Earth. Now the *mean anomaly* (in the text simply 'anomaly') of the planet is $\angle A_mHS$, measured in the direction of the motion on the epicycle. It is a linear function of time and can be found from the tables of mean motion. The *true anomaly* is $\angle A_vHS$, and can be found by adding $\angle A_vHA_m$ (which equals $\angle EHG$, i.e., the equation of centre) to, or subtracting it from, the mean anomaly. Note that the equation of centre must be added to the mean anomaly if the epicycle is located between the apogee and the perigee of the deferent, and subtracted otherwise. Consequently, the equation of centre must be added to the mean anomaly if it is subtracted from the mean centrum, and the other way around.

Next, the 'corrected centrum', the angle $\angle AES$ (measured from the apogee of the deferent) under which the planet is seen from the Earth is found by adding $\angle HES$, the *equation of anomaly*, to, or subtracting it from, the true centrum. Note that the equation must be added to the true centrum if the planet is located between the apogee and the perigee of the epicycle, i.e., if the true anomaly lies between 0° and 180°, and subtracted otherwise. Finally, the 'longitude' (*ṭūl*) or 'true longitude' (*taqwīm*) of the planet is the angle $\angle\Upsilon ES$ under which the planet is seen from the Earth, but measured from the vernal equinoctial point ♈. Thus the true longitude is found from the corrected centrum by adding the longitude of the apogee, $\angle\Upsilon EA$. Note that the apogees of the Sun and the five planets were assumed by Islamic astronomers to move slowly with respect to the equinoxes at a speed of one degree in approximately 66, $66\frac{2}{3}$ or 70 Persian years of 365 days (in the *Īlkhānī Zīj*, the apogee motion was taken equal to one degree in seventy Persian years).

In order to allow a rapid calculation of planetary positions for arbitrary points in time, Ptolemy and most Islamic astronomers provided a set of mathematical tables for the mean motions and the equations of each planet. While the mean motions are linear functions of time, whose tabulation is only complicated by the structure of the calendar used, the equations are highly complex trigonometrical functions whose calculation requires various look-ups in sine tables, calculations of square roots, multiplications and divisions. From the tables of mean motion, values are usually taken for the given numbers of years, months, days, and hours, which then need to be added. The equation of centre or 'first equation' is tabulated as a function of the mean centrum and can be read off directly or is found by carrying out linear interpolation between two consecutive values in the table.

However, the equation of anomaly is a function of both the true centrum and the true anomaly. Rather than providing a huge double-argument table, Ptolemy tabulated three types of functions of a single argument, whose values must be combined in an ingenious approximate procedure now generally called 'Ptolemaic interpolation'. The 'second equation', i.e., the equation of anomaly for the case that the epicycle centre is at its average distance from the Earth, as well as the so-called 'difference' (*ikhtilāf*), the differences between the equation of anomaly at the average position of the epicycle centre and at one of the extreme positions (namely, the apogee or perigee of the deferent), are tabulated as a function of the true anomaly. Furthermore, interpolation coefficients (referred to as 'interpolation minutes', in the text *daqāʾiq-i ḥiṣaṣ*) are provided as a function of the true centrum, i.e., the actual position of the epicycle centre between its average position and one of the two extremes. These are multiplied by the 'difference' and are then added to, or subtracted from, the central equation of anomaly in order to obtain an approximation to the actual equation of anomaly. The interpolation minutes assume values between zero (for the value of the true centrum corresponding to the average position of the epicycle centre) and one (for the apogee and perigee of the deferent). In this way they produce the correct equation of anomaly for the average as well as for the two extreme positions of the epicycle centre, and intermediate values for other positions.

In the remainder of this section, I will analyse most of the calculations of planetary longitudes in the Horoscope of Iskandar (which are unfortunately incomplete, because one folio preceding fol. 5 is missing). In spite of the occurrence of scribal and computational mistakes, which often propagate through sequences of calculations, most of the data can be seen to have been accurately computed on the basis of the standard Ptolemaic planetary models. Furthermore, thanks to peculiarities of the tabular values that are used in the calculations (such as the displacements discussed below) and thanks to a comparison of the values themselves with the most plausible sources for the Horoscope, it can be convincingly established that the planetary positions were systematically calculated on the basis of the tables in Naṣīr al-Dīn al-Ṭūsī's *Īlkhānī Zīj* (*c.* 1270, written in Persian). As a matter of fact, this is a very plausible source, since it was the most popular astronomical handbook in the eastern Islamic world from the late thirteenth to the middle of the fifteenth century. It is extant in dozens of manuscripts, several of which are now also available online.[22] At the same time when ʿImād al-munajjim compiled Iskandar's Horoscope, Ghiyāth al-Dīn Jamshīd al-Kāshī (probably ʿImād's grandson, cf. footnote 17), was working on his improvement of the *Īlkhānī Zīj*, the *Zīj-i Khāqānī dar takmīl-i Zīj-i Īlkhānī*. Furthermore, in the two extant versions of the Horoscope of Iskandar's half-brother Rustam, which were compiled in 1419 and 1423, probably by relatives of ʿImād al-munajjim, and which are in many respects similar to the Horoscope of Iskandar, the planetary positions are explicitly indicated to have been calculated from the *Īlkhānī Zīj*.[23] All following examples of calculations from the Horoscope are reconstructed on the basis of the tables in the *Īlkhānī Zīj*, and occasional deviations from that work are explained.

Computational example: Calculation of the longitude of Venus at the time of 2v: 25–27
the nativity. The mean centrum of Venus is given in the text as $10^{s}\,19;18,25°$, which is precisely the value that can be obtained from the mean motion parameters of the *Īlkhānī Zīj* for the exact mean time of the nativity, 10;40,13 hours after noon on Sunday, 24 April 1384 (cf. p. 25).[24] The given mean anomaly, $7^{s}\,18;6,27°$, differs by only $10''$ from a recomputation on the basis of the *Īlkhānī Zīj*. With the above-mentioned value of the mean centrum the equation of centre can be found as 3;12,23° by carrying out a linear interpolation between al-Ṭūsī's tabular values 3;13° for mean centrum $10^{s}\,19°$ and 3;11° for $10^{s}\,20°$ (the exact outcome of this interpolation is 3;12,21,30°). Now the true centrum

[22] See, for example, Berlin, Staatsbibliothek Preußischer Kulturbesitz, Sprenger 1853 (https://www.qalamos.net/receive/DE1Book_manuscript_00014176); Tehran, Majlis Library, MS 181 (https://dlib.ical.ir/site/catalogue/1033436); Los Angeles, University of California in Los Angeles, Caro Minasian 1462 (https://digital.library.ucla.edu/ catalog/ark:/21198/zz000wrfbd); and Paris, Bibliothèque nationale de France, persan 163 (https://archivesetmanuscrits.bnf.fr/ark:/12148/ cc1013621). Besides to the Paris copy, I provide references to Cairo, Dār al-kutub, *mīqāt fārisī* 1, and to Istanbul, University Library, F 1418. This latter manuscript was likewise copied for Iskandar Sultan and is similarly beautifully executed.

[23] The Horoscope of Rustam is extant in the manuscripts San Marino, Huntington Library, HM 71897 (this was previously MS Persian 1 in the Burndy Library at MIT, see now https://catalog.huntington.org/record=b1788124), and Qom, Marʿashī Library, 9233. The explicit reference to the *Īlkhānī Zīj* appears respectively on fol. 13r and on fol. 15v. The Huntington Library manuscript, fol. 8r, also refers to the *Īlkhānī Zīj* in connection with the correction of the ascendant (cf. Section 14).

[24] The true time of the nativity, 11 pm, leads to a mean centrum of $10^{s}\,19;19,15°$. Cf. Appendix C.

is obtained as $10^s\,22;30,48°$ by adding the equation of centre to the mean centrum, and the true anomaly as $7^s\,14;54,4°$ by subtracting the equation from the mean anomaly.

Next the equation of anomaly is calculated as follows by means of Ptolemaic interpolation. The equation of anomaly for the case that the epicycle centre is at its average distance from the Earth is taken as a function of the true anomaly from the table of the 'second equation'. Since the tabular values for $7^s\,14;30°$ and for $7^s\,15;0°$ are both equal to $10^s\,14;1°$, no interpolation is needed in order to find this same value.[25] The difference in the equation of anomaly between the average position of the epicycle centre and its perigee is found from the table for the 'difference' (*ikhtilāf*), again with the true anomaly as the argument. The result is given in the Horoscope as 1;9,12°, although the table in the *Īlkhānī Zīj* displays 1;11° for $7^s\,14°$ and 1;10° for $7^s\,15°$ (the value in the text would have been obtained if the tabular value for $7^s\,15°$ were 1;9 instead of 1;10). Since the epicycle centre is neither at its average distance from the Earth nor in its apogee or perigee, we need a Ptolemaic interpolation coefficient (*daqāʾiq-i ḥiṣaṣ*) in order to find an approximation to the equation of anomaly for the actual epicycle position in between the average distance and the apogee. This coefficient, a function of the true centrum, is found from the *Īlkhānī Zīj* as 48′, in agreement with the text. Now we multiply the coefficient by the 'difference' to obtain 0;55,21°. Following the rules in the *Īlkhānī Zīj*, this amount needs to be added to the second equation, yielding the 'adjusted equation' as $10^s\,14;56,21°$.

The equation of anomaly is then added to the true centrum in order to obtain the 'corrected centrum', i.e., the apparent position of Venus measured from the apogee of its deferent, as $9^s\,7;27,9°$. By adding the longitude of the apogee ('farthest distance') found in the text, $2^s\,19;50,47°$, likewise in agreement with the *Īlkhānī Zīj*, we finally find the true longitude of Venus as $11^s\,27;17,56°$.

The models for the Sun, Moon and Mercury differ to some extent from the general planetary model described above.[26] The Sun moves uniformly on an eccentric deferent (or, equivalently, it moves uniformly on an epicycle whose centre moves at the same angular velocity as the Sun on a circle around the Earth). The Moon moves in clockwise direction on its epicycle, while the centre of its deferent moves clockwise on a small circle around the Earth. In the Mercury model, the centre of

[25] The equation of anomaly for Venus as given in the Horoscope is not actually of the order of 10 zodiacal signs, but was 'displaced' by 12 signs (cf. Appendix B). Therefore the second equation given in the text is in fact a subtractive value of $1^s\,15;59°$. Because of the use of this displacement, the product of difference and interpolation minutes that is found below must be added to the second equation, whereas normally it would be subtracted.

[26] See Toomer, *Ptolemy's Almagest*, Books III–V and IX; Neugebauer, *HAMA*, Book I, Sections B and C3; Pedersen, *A Survey*, Chapters 5, 6 and 10, pp. 309–328; and Van Brummelen, *Mathematical Tables*, Chapters 8, 9 and 12, pp. 243–312.

the deferent moves on a small circle around a point twice as far removed from the Earth as the point of uniform motion and with a radius equal to the eccentricity; this will be explained in more detail in Section 12 as part of my attempt to explain the confused diagram of the orbs of Mercury on fol. 5v of the manuscript of the Horoscope. The exact details of the differences between the models do not concern us here. It suffices to know that the Sun has only a single equation, and that the rules for adding and subtracting the two lunar equations are somewhat different from those for the planets. In spite of the differences in the Mercury model, the use of its tables is identical to that for the other planets.

For practical calculations Islamic astronomers left Ptolemy's planetary models basically unchanged. However, they systematically improved the underlying mean motion parameters on the basis of fresh observations and incidentally also made adjustments to the eccentricities and epicycle radii. They continued a development already started by Ptolemy in his *Handy Tables* to make the tables for calculating planetary positions more convenient to use, for example by applying so-called 'displaced equations' (see below). Finally, in particular during the thirteenth century and later, various alternative planetary models were designed with the purpose of removing the non-uniform motions that result from making the motion of the epicycle centre uniform around a point different from the centre of the deferent (see also Section 12).

7.2. Recomputation of the Planetary Longitudes in the Horoscope

The planetary data in the Horoscope of Iskandar have certain characteristics that are typical for the source that was used for their calculation. These include, in particular, the displacements, which make the planetary equations always additive or always subtractive instead of additive for one range of arguments and subtractive for another, and a further development of displacements, the 'mixed equations'. In this commentary these two concepts will not be explained in every detail, but Appendix B summarizes some of the specific properties of the displacements of the planetary tables used for calculating the Horoscope and thus confirms the *Īlkhānī Zīj* as the most likely source of the planetary data in the Horoscope.[27] In fact, the planetary equations used in the Horoscope can

[27] A general discussion of displaced equations may be found in van Dalen, 'The *Zīj-i Nāṣirī*', pp. 840–841, and van Dalen, *Ptolemaic Tradition*, pp. 408–410. Studies of displaced tables are included in Salam & Kennedy, 'Solar and Lunar Tables'; Saliba, 'The Double-Argument Lunar Tables'; Saliba, 'Computational Techniques'; Saliba, 'The Planetary Tables', and, for the medieval Latin tradition, Chabás & Goldstein, 'Displaced Tables'.

be seen to be displaced precisely by the amounts also used in al-Ṭūsī's *Īlkhānī Zīj*, and to be of mixed type precisely where al-Ṭūsī uses equations of mixed type.

16r: 1–22 In particular for the time of conception (calculated in the text as
1;35,10,52 hours after midnight on July 13, 1383; cf. Section 14) the results of a recomputation on the basis of the *Īlkhānī Zīj* turn out to be remarkable: most of the mean motions listed in the Horoscope differ by at most one unit in the final sexagesimal digit (seconds for the Sun and the Moon, minutes for the five planets) from positions calculated on the basis of al-Ṭūsī's parameters and a longitude difference between Maragha and Uzgand of 20;50° (cf. Section 4). The small deviations can be easily explained from minor inaccuracies in al-Ṭūsī's mean motion tables and from the rounding errors in the four or five tabular values that need to be added in each case. Inaccurate mean motion values for
16r: 11 the time of conception are: the mean centrum of Mars (text: 8^s 19;59°,
16r: 16 recomputation 9^s 19;59°), the mean anomaly of Venus (text: 1^s 21;52°,
16r: 19–20 recomputation: 1^s 21;15°), and the mean anomaly of Mercury (found
from the first equation and the adjusted anomaly in the text as 11^s 6;0°,
16r: 3 recomputation: 11^s 7;44°). The mean lunar centrum is accurate if we
assume that a correction of 0;30° was added to the value for the mean lunar longitude found from the *Īlkhānī Zīj*. This addition is explicitly mentioned in the calculation of the lunar longitude at the time of conception, but was also applied in the calculation for the time of the nativity (both calculations appear in the context of Hermes's method for the correction of the ascendant, see Section 14). This correction, together with corrections for the other planets, were in fact suggested by other Islamic scholars only a short time after the compilation of the *Īlkhānī Zīj*, for instance by Quṭb al-Dīn al-Shīrāzī.[28]

The mean motions given in the Horoscope for the time of the nativity are somewhat more difficult to verify for various reasons. Firstly, as was noted above, only the last part of the calculation of Mercury's true longitude is included in what is left of the section on the true planetary positions at the time of the nativity (top of fol. 5r). Secondly, the solar
2v: 12–14 position is first calculated for 11 pm, the *true time* of the nativity, and
then adjusted for the corresponding *mean time* by subtracting the equation of time,[29] whereas the positions of all other planets are calculated

[28] See, for example, the introduction of the *Zīj-i Jāmiʿ-i Bū Sāʿidī* by Rukn al-Dīn al-Āmulī (*c.* 1455), Tehran, University Library, MS 5772, fols 1v–2r. These corrections were studied in Savadi & Nikfahm Khubravan, 'Ḥarakat-i wasaṭ-i kawākib', p. 373, with an edition of al-Āmulī's introduction on pp. 465–468.

[29] For a brief explanation of these concepts, see Appendix C.

directly for the mean time of the nativity. From the general table for
the equation of time in the *Īlkhānī Zīj*[30] we find for the true solar lon-
gitude at the true time of the nativity (1^{s} 12;39,35°) an equation equal 2v: 13
to 0;19,47 hours, implying that the mean time of the nativity was close
to 10;40,13 hours after noon. It is for this time that the mean motions
of the Moon and Venus, and therefore probably also those of the other 4r: 9–14
planets, can be seen to have been calculated. Of the total of nine mean 2v: 25–27
motions for the time of the nativity given in the extant part of the Horo-
scope, only the lunar anomaly contains a large scribal error (0^{s} 19;0,52°
instead of 0^{s} 19;5,52°), but all others agree with the *Īlkhānī Zīj* to within
an accuracy of 10″.

Also the solar, lunar and planetary equations given in the Horoscope
can be satisfactorily recomputed from the tables in the *Īlkhānī Zīj*. In
some cases, linear interpolation between two tabular values yields the
equations in the text to a precision of seconds (it can be verified that
the mean motions used as arguments for the interpolation were often
rounded or truncated to minutes); in other cases small errors, and occa-
sionally somewhat larger ones, were made. For instance, the given solar
equation values for the time of the nativity (3;29,58°) and for midnight 2v: 12
of the day of conception (1;6,5°) are accurate to seconds, but the value 4r: 18
for the corrected time of conception (1;5,44°) contains an error of 14″ 16r: 2
(correct: 1;5,58°). The lunar equations of centre at the times of con- 16r: 3
ception (3;35,37°) and birth (24;52,30°) are correct to seconds except 4r: 10
for one common scribal mistake (37″ for 32″). The lunar equation of
anomaly, as well as the 'equation difference', the interpolation minutes,
and the correction for the inclination of the lunar orbit (the so-called
'third equation') are all accurate to within one or two seconds.

For the other planets, the typical error in the longitudes is of the order
of one minute, obviously because that is the precision of the tables for
the planetary equation of centre and equation of anomaly. A large error
occurs in the longitude of Mars at the time of conception because of the
above-mentioned mistake of a whole zodiacal sign in the mean centrum.
It can be verified that the equations of Mars were taken consistently with
the wrong centrum; the correct longitude would have been near 3^{s} 9;18°.
A smaller error is found in the calculation of the longitude of Venus at
the time of conception: the equation of centre should have been 0;47°
instead of 1;47°, so that also the longitude becomes roughly one degree 16r: 16
too large. An overview of all planetary positions given in the Horoscope
is provided in Appendix F.

[30] For instance, Paris, Bibliothèque nationale de France, persan 163, fol. 123r; Cairo, Dār al-kutub, *mīqāt fārisī* 1, fol. 122r; or Istanbul, University Library, F 1418, fol. 237v.

8. Planetary Latitudes

The Ptolemaic models for the planetary longitudes were devised under the assumption that the planets move in the plane of the ecliptic. However, the Moon and the five planets visible to the naked eye can easily be observed to periodically move above and below the ecliptic by an amount of up to around 8 degrees. The orthogonal distance of any heavenly body from the ecliptic is called its 'latitude' (*ʿarḍ*). The lunar latitude can simply be modeled by giving the plane of the lunar motion an inclination of approximately five degrees to the ecliptic. The small correction to the lunar longitude that is necessary because of this inclination was tabulated in many Islamic zījes under the name 'third equation'. The latitude theory for the five planets, which would be relatively simple in a heliocentric system but becomes highly complicated in a geocentrical model, is expounded in the last book of the *Almagest*. By using another type of Ptolemaic interpolation, Ptolemy manages to reduce the complicated oscillatory motions of the deferent and the epicycle to a simple arithmetical solution with only three single-argument tables for each planet. As was shown in van Dalen, 'Tables of Planetary Latitude', the latitude tables in Islamic zījes generally stayed close to the *Almagest*. The setup of the tables was slightly modified in order to make them easier to use, but in only very few cases were the underlying parameter values based on new observations.[31]

Following the defective section on planetary longitudes in the Horo-
5r: 9–29 scope of Iskandar Sultan, the latitudes of the Moon and the five planets at the time of the nativity are calculated step by step. In order to reproduce the calculations, I will now briefly explain the technical characteristics of Ptolemy's models for planetary latitude.[32] For the Moon as well as the planets, the points of intersection of the inclined deferent with the plane of the ecliptic are called 'nodes' (*jawzahar*); the 'ascending node' (*raʾs*, lit. 'Head') is the point of intersection where the lunar or planetary epicycle centre passes the ecliptic from south to north, and the 'descending node' (*dhanab*, lit. 'Tail') is the point where the epicycle centre passes the ecliptic from north to south.

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[31] For a discussion of some of the Islamic observational programs that led to new parameters for planetary latitudes, see Mozaffari, 'Planetary Latitudes'.

[32] More extensive explanations of the calculation of planetary latitudes according to Ptolemy can be found, for example, in Toomer, *Ptolemy's Almagest*, Book XIII; Neugebauer, *HAMA*, Book I, Section C7, pp. 206–230; Pedersen, *A Survey*, Chapter 12; Van Brummelen, *Mathematical Tables*, Chapter 14, pp. 338–373, and Van Brummelen, 'The Tables of Planetary Latitudes'.

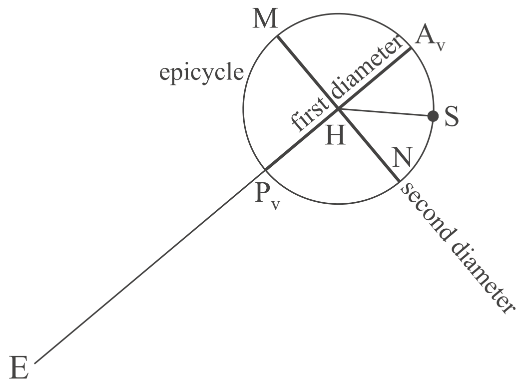

Figure 4: Definition of the first and second diameter of the epicycle

We first refer to Figure 4, which depicts a planetary epicycle with centre H. In this figure, E represents the Earth, S the planet, A_v the true apogee, and P_v the true perigee of the epicycle (cf. Section 7 and Figure 3). The *first diameter* of the epicycle, P_vA_v, is its intersection with the plane perpendicular to the deferent that passes through the epicycle centre and the Earth. The *second diameter* of the epicycle, MN, is perpendicular to the first.

In Ptolemy's model for the latitude of the superior planets, the deferent is given a constant inclination to the plane of the ecliptic. Furthermore, the epicycle receives a small oscillatory motion around its second diameter (i.e., both A_v and P_v move alternately above and below the plane of the deferent). Instead of carrying out the complicated calculation of the resulting latitude for each pair of values for the true centrum and the true anomaly, Ptolemy (and after him most Islamic astronomers) again applies Ptolemaic interpolation: he tabulates the latitude factor caused by the oscillation of the epicycle as a function of the true anomaly for both the northernmost and the southermost position of the epicycle centre on the deferent, and then multiplies the result by interpolation coefficients depending on the position of the epicycle centre, i.e., on the true centrum. Similar to the interpolation function for the equation of anomaly (cf. Section 7.1), the latitude interpolation coefficients are equal to zero when the epicycle centre is in one of the nodes and equal to one when the epicycle centre is in the northernmost or southernmost point of the deferent.

In the case of the inferior planets Venus and Mercury (see Figure 5), also the deferent has a small oscillatory motion around the ecliptic plane, called the *inclination* (Arabic *mayl al-falak al-khārij al-markaz*). This motion has the same periodicity as that of the epicycle centre H on the

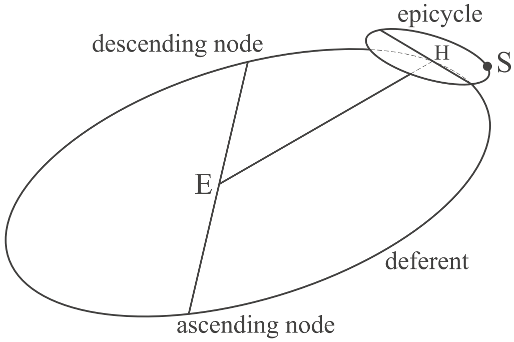

Figure 5: Model for the latitude of the inferior planets

deferent in such a way that the epicycle centre of Venus is always north of the ecliptic and that of Mercury always south. The epicycle of the inferior planets oscillates around its second diameter (the *deviation*, Arabic *mayl*) as well as around its first diameter (the *slant*, Arabic *inḥirāf*). These oscillations depend on the position of the epicycle centre on the deferent in such a way that the deviation reaches its maximum and the slant becomes zero when the epicycle centre is in one of the two nodes, while the slant assumes its maximum and the deviation is zero when the inclination of the deferent reaches one of its two maximum values (at these moments the centre of the epicycle is at the apogee or perigee of the deferent, which, only for the inferior planets, are precisely 90° removed from the nodes). Since Ptolemy assumes that the three latitude components of the inferior planets can be treated independently, the calculation of the latitudes becomes relatively simple.[33] The 'first latitude', due to the inclination, is usually tabulated directly as a function of the true centrum. The 'second latitude', due to the deviation, and the 'third latitude', due to the slant, are both found by means of Ptolemaic interpolation as the product of a function of the true anomaly and an interpolation function of the true centrum. In the case of Mercury, an extra addition or subtraction of a tenth of the obtained value for the slant is carried out depending on whether the epicycle centre is located in the northern or southern half of the deferent.

By comparing the calculations of the lunar and planetary latitudes presented in the Horoscope of Iskandar with those expounded by Ptolemy in the *Almagest*, we can recognize the following characteristics of the underlying planetary tables:

[33] One of only very few Islamic authors who attempted a more exact calculation of planetary latitudes was the grandson of the author of Iskandar's Horoscope, Ghiyāth al-Dīn Jamshīd al-Kāshī; see Van Brummelen, 'Taking Latitude with Ptolemy'.

1] *Moon*. The underlying maximum lunar latitude can be shown to be precisely equal to 5°: If β denotes the lunar latitude, $\beta_{\max}$ its maximum value, and λ_n the latitude argument (*ḥiṣṣa-yi ʿarḍ*, the distance of the Moon from the ascending lunar node), we have $\sin\beta(\lambda_n) = \sin\beta_{\max} \cdot \sin\lambda_n$. Since the text gives the argument of latitude as $9^{\mathrm{s}}\,18;16,6°$ and from this value finds the latitude as 4;44,52° south, it follows that $\beta_{\max} = \arcsin(\sin\beta / \sin\lambda_n) \approx \arcsin(\sin -4;44,52° / \sin 288;16,6°) \approx 5;0,1$. It can be verified that linear interpolation between arguments $9^{\mathrm{s}}\,18°$ and $9^{\mathrm{s}}\,19°$ in the lunar latitude table in the *Īlkhānī Zīj*, which has 5;0,0 as its maximum, produces exactly the value given in the text. 5r: 9–10

2] *Superior planets*. In the text an addition of 7° is carried out to find the 'actual adjusted centrum' (*markaz-i muʿaddal-i ḥaqīqī*) of Saturn from its 'adjusted centrum' (*markaz-i muʿaddal*) before the latitude is calculated. For Jupiter, a similar addition of 12° is performed. These two adjustments are indications of the displacements of 7° and 12° in the respective equations of anomaly (cf. Appendix B). As we have seen above, the latitudes of the superior planets are generally found by multiplying the maximum northern or southern latitude (in the text 'latitude', *ʿarḍ*; for Saturn mistakenly 'argument of the latitude', *ḥiṣṣa-yi ʿarḍ*) for the calculated value of the true anomaly by interpolation minutes (*daqāʾiq-i ḥiṣaṣ-i ʿarḍ*) that depend on the position of the epicycle centre on the deferent. This position of the epicycle centre can be expressed by either the true centrum (its distance from the apogee of the planet) or the latitude argument (the distance of the epicycle centre from the ascending node). From the latitude calculations in the Horoscope of Iskandar it can be noted that, in the zīj that was used, the interpolation minutes were tabulated as a function of the true centrum rather than of the latitude argument, since no conversion of the former to the latter is carried out. The latitude tables for the superior planets in the *Īlkhānī Zīj* indeed have the true centrum rather than the latitude argument as their arguments. 5r: 11–19

Linear interpolation in al-Ṭūsī's tables for Saturn leads to an interpolation constant 0;31,51 (text: 0;31,50) for the indicated true centrum $6^{\mathrm{s}}\,8;9°$, and to a maximum southern latitude of 2;9,41° (text: 2;9,40°) for the indicated true anomaly $10^{\mathrm{s}}\,21;56°$ (note that the *Īlkhānī Zīj* tabulates the maximum northern and southern latitudes for every six degrees only). For Jupiter, the maximum southern latitude for the given true anomaly $11^{\mathrm{s}}\,4;38°$ should have been 1;7,14° instead of the text's 1;8,46° (this mistake could be explained by assuming that the linear interpolation was carried out between the values 1;8° and 1;9° for arguments $10^{\mathrm{s}}\,24°$ and $11^{\mathrm{s}}\,0°$ rather than between the values 1;8° and 1;7° for arguments $11^{\mathrm{s}}\,0°$ and $11^{\mathrm{s}}\,6°$). For Mars, the interpolation minutes found in the text are in full agreement with al-Ṭūsī. The given maximum northern latitude, 4;17,40°, can be obtained for a true anomaly of 183;10° (the true anomaly of Mars at the time of the nativity, which is not mentioned in the extant part of the Horoscope, can be calculated from the *Īlkhānī Zīj* as 183;14°).

5r: 19–29 3] *Inferior planets*. Of the three latitude components of Venus and Mercury, the inclination is taken directly from the table for the 'first equation', while the deviation is found as the product of the 'argument of the second latitude' and the corresponding latitude interpolation function (*daqāʾiq-i ḥiṣaṣ-i ʿarḍ-i thānī*), and the slant as the product of the 'argument of the third latitude' and an interpolation function. The values of the three components as given in the Horoscope of Iskandar can be accurately reproduced from the tables in the *Īlkhānī Zīj*. For Mercury, no explicit increase or decrease of the third latitude (*slant*) by a tenth (as in the *Almagest* and early Islamic zījes) is indicated in the text. Also this characteristic is in agreement with the *Īlkhānī Zīj*, which has incorporated the tenth in the table for the third latitude. By doing a reverse lookup in al-Ṭūsī's table with the second latitude 2;48,46° and the third latitude 2;10,50° arrived at in the Horoscope, we find that the underlying true anomaly must have been 144;35°, in full agreement with the second equation given in the section on longitudes (fol. 5r: 1) and with a recomputation based on the parameters of the *Īlkhānī Zīj*.

9. Direct and Retrograde Motion

4v: 4–6 At more or less regular intervals of time the five planets change the direction of their motion in the usual order of the zodiacal signs to become retrograde. In the planetary model described in Section 7 (see Figure 3 on p. 17), this occurs when the planet moves on the part of its epicycle nearest to the Earth and its velocity on the epicycle results in a backward component of its total motion larger than that contributed by the motion in centrum (i.e., by the motion of the epicycle centre around the deferent). As will be indicated in Section 12 for the particular case of Mercury, Apollonius (3th c. BCE) already formulated criteria whether a planet is in direct or in retrograde motion in the case of a simple epicycle model. Ptolemy adjusted these to his more complex planetary models and included in the *Almagest* tables for the so-called 'stations' (*maqām*, pl. *maqāmāt*), the points in time when the planets change from direct motion (*istiqāmat*) to retrograde motion (*rujūʿ* or *rujʿa*) or vice versa.[34]

[34] The theory of Apollonius, its application to the planetary models by Ptolemy, and the tables for the planetary stations found in the *Almagest* (see Toomer, *Ptolemy's Al-*

These tables, which were adopted by many Islamic astronomers and were only subject to small changes in the course of time, provide for any given value of the mean centrum the value of the true anomaly for which a planet reaches one of its stations. Also al-Ṭūsī copied Ptolemy's tables into the *Īlkhānī Zīj* with only minor differences that are mostly due to scribal mistakes.[35] As an example, we will determine whether Mercury was in direct or in retrograde motion at the time of the nativity (see also Section 12).

Computational example: Determination of the phase of Mercury at the time of 3r: 3–4
birth of Iskandar Sultan. The true centrum of Mercury at the time of Iskandar's birth is given in the Horoscope as $6^{s}\,8;18,13°$ (fol. 5r: 1). The table of the first station of Mercury in the *Īlkhānī Zīj* displays the values 144;40° for true centrum $5^{s}\,24°$ or $6^{s}\,6°$, and 144;39° for true centrum $5^{s}\,18°$ or $6^{s}\,12°$. By carrying out a linear interpolation between these values, we find that, for the above value of the true centrum, Mercury reaches its first station for a true anomaly of 144;39,37°, and hence its second station for a true anomaly of 215;20,23° (the true anomalies for the first and second stations add up to 360°). Thus Mercury is in retrograde motion if, for the above value of the mean centrum, its true anomaly is between 144;39,37° and 215;20,23°. Since the true anomaly at the time of Iskandar's birth can be reconstructed from the values of the second and third latitudes as 144;35° (see Section 8), we can confirm the statement found in the Horoscope that Mercury will soon start its retrograde motion (we find the difference in true anomaly to be $4\frac{1}{2}'$, whereas the text gives $3'$).

In a similar way it can be confirmed that of the other planets only Mars was 5v: 4–6
in retrograde motion at the time of the nativity, in accordance with the text of the Horoscope.

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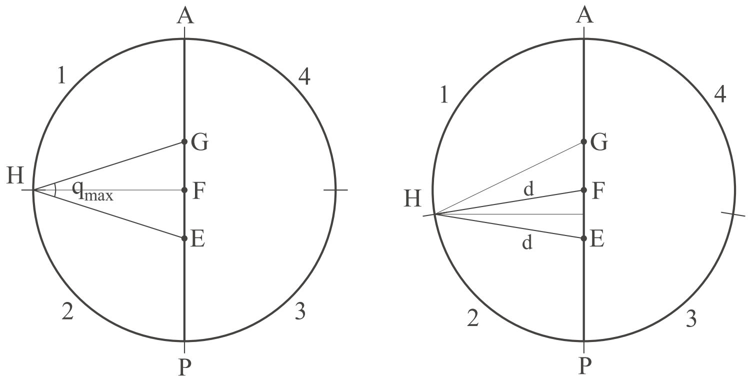
magest, Book XII, Sections 1–8), are explained in Neugebauer, *HAMA*, Book I, Section C6, pp. 183–206; Pedersen, *A Survey*, Chapter 11; and Van Brummelen, *Mathematical Tables*, Chapter 13, pp. 314–326.

[35] Al-Ṭūsī's table for the first station of the five planets (together with his tables for the planetary sectors and planetary visibility) is found in Paris, Bibliothèque nationale de France, persan 163, fol. 57v, and Istanbul, University Library, F 1418, fol. 173v. The explanatory text in the second part of Section 5 of Treatise 2 is found on folios 22r and 126r, respectively. Al-Ṭūsī prescribes that the tables are to be used with the true centrum rather than with the mean centrum, but does not seem to have modified Ptolemy's tables to account for this difference. He does not explicitly state that the 'actual adjusted centrum' (i.e., the true centrum corrected for the displacement of the equation of centre, cf. Section 8) must be used, but he does add the displacements to the true centrum when calculating the latitudes of Saturn and Jupiter. Note that in the *Handy Tables* Ptolemy expressed the stations in terms of the mean anomaly rather than the true anomaly, which led to clear differences from the tables in the *Almagest*. Further research is necessary to establish whether al-Ṭūsī made a slip of the pen in his instructions or deliberately used the *Almagest* tables with the true centrum as the argument.



a) ‘according to motion’ b) ‘according to distance’

Figure 6: The sectors of the planetary deferents

10. Sectors of the Planets

5v: 1–4 The eccentric circle of the Sun and the deferents and epicycles of the Moon and the five planets are each divided into four *sectors* (*niṭāq*, pl. *niṭāqāt*). The sectors of the eccentre or deferent (*falak-i awj*) are referred to as *awjī*, lit. ‘of the apogee’, and those of the epicycle (*falak-i tadwīr*) as *tadwīrī*. Planetary sectors were used by Abū Maʿshar as early as the ninth century and were tabulated in zījes by al-Bīrūnī and many later astronomers.[36] Naṣīr al-Dīn al-Ṭūsī defines the sectors in the following way in Treatise 2, Section 5 of his *Īlkhānī Zīj*.[37] His definitions are extremely brief; all explanations between square brackets have been added by the present author.

For each planet the beginning of the first sector of the deferent is the apogee (*awj*, point *A* in Figure 6), and the beginning of the third sector the perigee (*ḥaḍīḍ*, point *P*). The beginnings of the second and fourth sectors can be defined in two ways, ‘according to motion’ (*bi ḥasb-i sayr* or *bi ḥasb-i ḥarakat*) and ‘according to distance’ (*bi ḥasb-i buʿd*). In order to explain these two concepts we need the following points in the figure: *E* is the Earth (or, equivalently, the centre of the universe), *F* the centre of the deferent, and *G* the centre of uniform motion. In both cases, *H* indicates the beginning of the second sector.

[36] See, for instance, Saffouri et al., *Al-Bīrūnī on Transits*, pp. 60–72 (with commentary on pp. 161–166), and the study thereof in Kennedy, ‘The Sasanian Astronomical Handbook’, pp. 247–253.

[37] See, for example, Paris, Bibliothèque nationale de France, persan 163, fols 21v–22r; Cairo, Dār al-kutub, *mīqāt fārisī* 1, fols 27v–28r; or Istanbul, University Library, F 1418, fol. 126r.

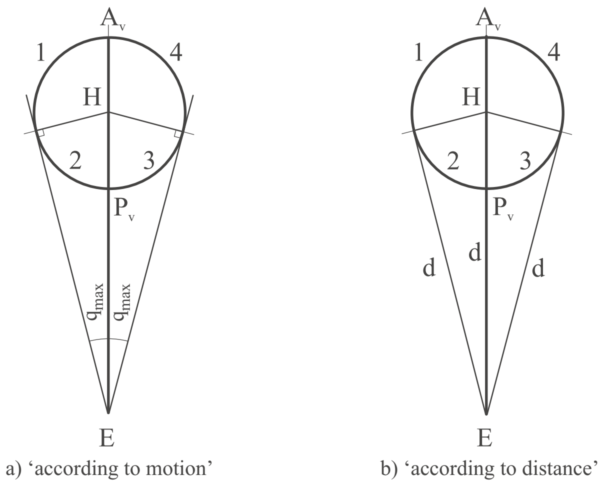


Figure 7: The sectors of the planetary epicycles

In the first case, the beginnings of the second and fourth sectors are the points where the motion of the centre of the epicycle is 'neither quick nor slow'. [This means that the motion assumes its mean value, and hence that the equation of centre reaches its maximum q_{max}. This occurs when the epicycle centre lies on the line through F perpendicular to AP (cf. Figure 6a) or, equivalently, when the mean centrum equals $90° + \frac{1}{2}q_{max}$ and the true centrum $90° - \frac{1}{2}q_{max}$, or 360° minus these values.] In the case of sectors defined 'according to distance' (see Figure 6b), the beginnings of the second and fourth sectors are the points where the distance of the epicycle centre H from the centre of the universe (i.e., the Earth E) is equal to its distance from the centre of the deferent F (i.e., to the radius of the deferent). [This happens when the epicycle centre lies on the line through the middle of EF and perpendicular to AP.][38]

The beginning of the first sector of the epicycle of any planet is its true apogee (*dhirwa*, A_v in Figure 7), and the beginning of the third sector its true perigee (*ḥaḍīḍ*, P_v). The beginning of the second and fourth sectors defined 'according to motion' are the points where the planet has 'only motion in centrum'. [These are the points where the equation of anomaly does not change, i.e., where it reaches its maximum value. They can be found as the points where the tangents extending from the centre of the universe (E) touch the epicycle.] Defined 'according to

[38] The same definitions are valid for the plain eccentric model of the Sun and for the slightly more complicated models for the Moon and Mercury. However, for the latter two the actual calculations of the beginnings of the second and fourth sectors are more complex.

distance', the second and fourth sectors are the points of the epicycle where the distance of the planet from the Earth is equal to the distance of the epicycle centre from the Earth.

Planets in the first and second sectors of the eccentric circle or the epicycle are called 'descending' (*hābiṭ*), while those in the third and fourth sectors are called 'ascending' (*ṣāʿid*). According to the *Īlkhānī Zīj*, planets in the fourth and first sectors are also called 'high' (*mustaʿlī*) and those in the second and third sectors 'low' (*munkhafaḍ*), but these terms are not used in the Horoscope of Iskandar.

The *Īlkhānī Zīj* provides a table displaying for each planet the beginnings of the sectors of the eccentric circle and the epicycle expressed in values of the true centrum and the true anomaly respectively.[39]

5v: 1–4 It can be verified that all indications of sectors and ascendance or descendance given in the Horoscope of Iskandar for the time of his birth are correct, except that Venus is in the fourth sector of its eccentre and in the third sector of its epicycle rather than the other way around. When using the tables for the sectors in the *Īlkhānī Zīj*, one needs to take into account that the true ('adjusted') centrums of Saturn and Jupiter as given in the text are displaced by 7° and 12° respectively. As we have seen in Section 8, the 'actual adjusted centrums' need to be determined when one calculates the planetary latitudes, and likewise for determining the sectors and the stations (cf. Section 9).

11. Comets

5v: 7 Many zījes contain a table for the motion of the comet Kayd together with positions at a given epoch for six or seven different comets (*dhawāt-i adhnāb*, lit. '[stars] having tails'). The *Īlkhānī Zīj* does not contain such a table, but the contemporary zīj of Jamāl al-Dīn Abū al-Qāsim ibn Maḥfūẓ al-munajjim al-Baghdādī, written in 1285/86 and extant in a unique manuscript in Paris, provides complete information on the positions and the motions of the same seven comets that are also mentioned in the Horoscope.[40] In nearly all zījes the motion of the comets amounts to exactly $-2°30'$ in a Persian year, the minus indicating that,

[39] See, for example, Paris, Bibliothèque nationale de France, persan 163, fol. 57v, and Istanbul, University Library, F 1418, fol. 173r. The table was reproduced in Kennedy, 'The Sasanian Astronomical Handbook', pp. 250–252, where it was already noticed to be very inaccurate and of a structure different from al-Bīrūnī's tables.

[40] See Paris, Bibliothèque nationale de France, arabe 2486, fols 98v–100r. This material was analysed in Kennedy, 'Comets', pp. 48–49. Kennedy transliterates the names of the comets as follows: Ghaṭayṭ, ʿAẓīm, Sarmūs or Sar-i Mūsh, Kilāb, Laḥyān, Dhū al-Dhawāba, and Kayd.

as in the case of the lunar nodes, the motion takes place in the opposite direction of the signs of the zodiac. From al-Baghdādī's positions for the beginning of the year 631 Yazdigird (10 January 1262), reproduced by Kennedy, we find the following positions for the time of birth of Iskandar Sultan (with the names of the comets spelled as in the Horoscope): Ghaṭīẓ 1^{s} 9;49°, Gharīm 11^{s} 28;35°, Sar-i Mūsh 6^{s} 24;7°, Kalāb 10^{s} 6;39°, Laḥyānī 10^{s} 6;48°, Dū dhawāba 10^{s} 4;5°, Kayd 4^{s} 23;28,4°. In the text of the Horoscope the places for the longitudes were all left blank, and in the table with positions for the time of birth on fols 16v–17r the comets are not included.

12. The Configuration of Mercury

A coloured diagram on fol. 5v of the Horoscope (see Figure 8) shows 5v
the configuration of the orbs of Mercury at the time of the nativity. Although this time may have been favourable for the native, it was not particularly convenient for drawing the configuration of Mercury, since many of the lines in the diagram almost coincide. The interpretation of the diagram is made even more difficult by the fact that the letters indicating points, circles and arcs were omitted from the manuscript, and that some of the orbs seem to have been drawn incorrectly. Figure 9 reproduces the configuration depicted in the manuscript with the mean centrum increased from 6^{s} 7;53° to 7^{s} 0° in order to improve clarity. The letters have been inserted as adequately as possible on the basis of the description in the text,[41] small question marks indicating that the placement is not unambiguously defined. This happens, in particular, in cases where a letter occurs only once in the definition of a circle (for example, points *Y* and *W*). The letters *thāʾ* (Θ in Figure 9), *dāl* (*D*), and *qāf* (*Q*) are used for two different points in the configuration, and the letter *ʿayn* (*O*) for three different points. These multiple occurrences are indicated by means of single and double primes. In some other cases where the same letter appeared to be used for multiple points, the ambiguity could be resolved by applying plausible corrections of scribal errors (in particular, confusion of *ṭāʾ* ط and *ẓāʾ* ظ and of *ʿayn* ع and *ghayn* غ; cf. footnote 41).

[41] The Arabic letters have been transcribed according to the system proposed by Kennedy and Hermelink; see Kennedy, 'Transcription'. Note that the following letters can easily be confused in the manuscripts because their Arabic equivalents differ only by diacritical dots: *B*, *U* and Θ (*bāʾ*, *tāʾ* and *thāʾ*); *G*, *H* and *J* (*jīm*, *ḥāʾ* and *khāʾ*); *D* and Φ (*dāl* and *dhāl*); *Z* and *R* (*zāʾ* and *rāʾ*); *T* and *V* (*ṭāʾ* and *ẓāʾ*); *S* and *X* (*sīn* and *shīn*); *O* and *I* (*ʿayn* and *ghayn*); *F* and *Q* (*fāʾ* and *qāf*); and *C* and Σ (*ṣād* and *ḍād*). From the length of this list it will be clear that identifying the letters in Arabic or Persian diagrams is often a nontrivial matter.

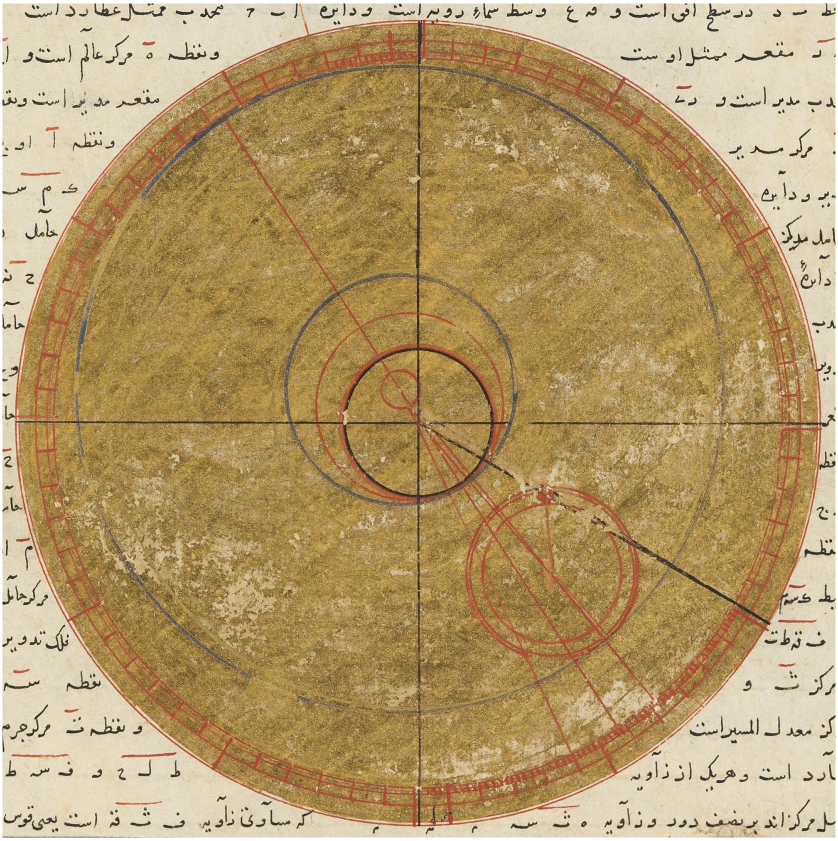


Figure 8: The configuration of Mercury as depicted in the manuscript of the Horoscope of Iskandar Sultan (London, Wellcome Library, Persian 474, fol. 5v)

As was mentioned in Section 7, Ptolemy's model for Mercury is somewhat different from that for the other four planets depicted in Figure 3 on page 17. Also Mercury (indicated by point *U* in Figure 9) moves with a uniform angular velocity and counter-clockwise on an epicycle, whose centre Θ moves with a uniform angular velocity and counter-clockwise on a deferent (in Figure 9 indicated by 'Ptolemaic deferent' for reasons that will be explained below). However, for Mercury the centre *M* of the deferent is not fixed: it moves with the same angular velocity as the epicycle centre but in opposite (i.e., clockwise) direction on a small circle *KMS* with centre *L*. This point *L* is a distance 2*e* removed from the Earth *E*, where *e* is Mercury's eccentricity, i.e., the distance of its centre of uniform motion *S* from the Earth. The radius of the small circle carrying the centre of the deferent is also equal to *e*, so that the distance of *M* from the Earth varies between *e* and 3*e*. It can be verified

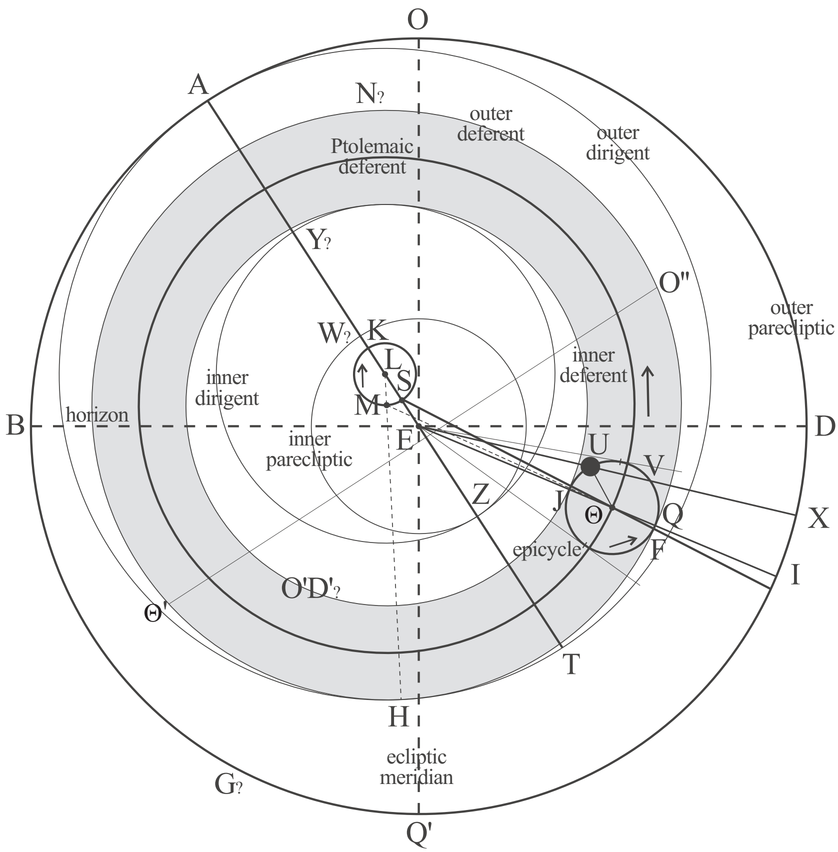


Figure 9: Reconstruction of the configuration of Mercury (with the mean centrum changed from $6^s\,7;53^\circ$ to $7^s\,0^\circ$ for greater clarity). Legend:

E	Earth ('centre of the world')	T	perigee of the dirigent
U	Mercury	H	'farthest point' of the deferent
BD	horizon	$FQVU$	epicycle
OQ'	ecliptic meridian circle	Θ	epicycle centre
ABG	outer parecliptic	F	mean apogee of epicycle
WZ	inner parecliptic	Q	true apogee of epicycle
L	centre of the dirigent and the small circle KMS	$\angle AS\Theta$	mean centrum
		$\angle AE\Theta$	true centrum (= arc ABI)
AHT	outer dirigent	$\angle E\Theta S$	equation of centre
ZY	inner dirigent	$\angle F\Theta U$	mean anomaly
M	deferent centre	$\angle Q\Theta U$	true anomaly
KMS	carrier of the deferent centre	$\angle \Theta EU$	equation of anomaly
HN	outer deferent	$\angle AEU$	'corrected centrum' (= arc $ABGIX$)
$O'D'$	inner deferent	V	intersection of line EU with the epicycle
S	centre of the equant	Θ'	beginning of the 2nd sector of the deferent
A	apogee of dirigent and deferent	O''	beginning of the 4th sector of the deferent

that, as a result of this model, Mercury reaches its nearest position to the Earth twice during each revolution rather than once as for the other planets.[42]

The diagram of the configuration of Mercury in the Horoscope of Iskandar Sultan basically depicts Ptolemy's model. However, ʿImād al-munajjim Maḥmūd al-Kāshī followed a cosmological implementation of the model that had been described by various astronomers in the thirteenth and fourteenth centuries, most prominently by Naṣīr al-Dīn al-Ṭūsī in his *Tadhkira fī ʿilm al-hayʾa* (*Memoir on Astronomy*, written between 1261 and 1274).[43] Rather than investigating of which source precisely ʿImād made use, I will only describe the relevant characteristics of the planetary models in the *Tadhkira* and discuss to which extent these can be recognized in the diagram in the Horoscope.

Already some of the earliest Islamic astronomers criticized Ptolemy's planetary models as found in the *Almagest* for the following two reasons. First, the fact that the centre of uniform motion of the epicycle centre of the five planets is different from the centre of their circular paths violates the principle of uniform motion on circles as formulated by Aristotle. Secondly, the mathematical models from the *Almagest* lack a physical interpretation in which the universe consists completely of solid bodies (a first step in this direction was in fact already made by Ptolemy himself in the *Planetary Hypotheses*). The *Tadhkira* was one of a number of works by Islamic scholars in which solutions for the so-called 'difficulties' (*ishkālāt*) related to non-uniform motion were sought and a physical basis for Ptolemy's models was given. Thus in the *Tadhkira* the circles representing the ecliptic and the planetary deferents and equants are replaced by orbs (*falak*, pl. *aflāk*) that are solid bodies with a certain thickness and are bounded by an inner and outer spherical surface. Furthermore, the non-uniform motion of the epicycle centres on the deferent is replaced by a combination of uniform motions (the so-called *Ṭūsī couple*) producing exactly the same positions of the planets as Ptolemy's models.[44]

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[42] More details of the Mercury model can be found in Toomer, *Ptolemy's Almagest*, Book IX, pp. 419–467; Neugebauer, *HAMA*, Book I, Section C3, pp. 158–169; Pedersen, *A Survey*, Chapter 10; and Van Brummelen, *Mathematical Tables*, Chapter 12, pp. 247–253.

[43] For an edition and translation of the *Tadhkira* with extensive commentary and ample background information, see Ragep, *Memoir on Astronomy*.

[44] The Ṭūsī couple is discussed in detail in Ragep, 'The Two Versions'. Other Islamic astronomers attempted to solve the 'difficulties' of Ptolemy's models by making different types of modifications, such as the introduction of additional epicycles. A particularly interesting example are the planetary models of Ibn al-Shāṭir (Damascus,

Both the diagram in the Horoscope of Iskandar and its description in the text make clear that a representation with solid orbs was intended. In fact, the text mentions an *inner* (‘concave’, *muqaʿʿar*) and *outer* (‘convex’, *muḥaddab*) parecliptic (*mumaththal*), dirigent (*mudīr*), and deferent (*ḥāmil*). Note that the parecliptic represents the ecliptic of Ptolemy’s plane planetary model, and the dirigent the motion of the deferent centre on a small eccentric circle (see below). The epicycle lies within the deferent, the deferent moves within the dirigent, and the dirigent orb lies fixed within the parecliptic. Although no inner and outer epicycles occur explicitly in the text, the diagram does show two circles for the epicycle that enclose the planet, which might give a hint as to the actual source used by the author. For somewhat greater clarity, in the reconstruction of the configuration in Figure 9, I have shaded the deferent orb, in whose middle lies the ‘Ptolemaic deferent’, i.e., the actual circular path of the epicycle centre, which is not referred to in the text of the Horoscope.

The explanation in the text starts by fixing the configuration of 5v: 10–27
Mercury with respect to the horizon. BD is the intersection of the (par)ecliptic plane with the horizon, and OQ', perpendicular to BD, that with the *ecliptic meridian circle* (‘midheaven circle of visibility’, *wasaṭ-i samāʾ-i ruʾyat*), i.e., the circle through the poles of the horizon and those of the ecliptic. The use of the letters *ʿayn* and *qāf* (O and Q) for the ecliptic meridian is not in agreement with the later occurrences of these letters. Circle ABG is the outer parecliptic, and WZ the inner parecliptic; both have the Earth E (‘the centre of the universe’) as centre. The circle AHT is the outer dirigent, and ZY the inner dirigent; both are circles around L, the centre of the small circle KMS that carries the centre of the deferent. Circles HN and $O'D'$ (the use of the letters O and D here is inconsistent with their previous and later occurrence[45]) are the outer and inner deferent, which both have point M on the small circle around

c. 1350), which were shown to be mathematically equivalent to those of Copernicus (*c.* 1500). More information on non-Ptolemaic planetary models in the Islamic world can be found, for example, in: Kennedy, ‘Late Medieval Planetary Theory’ and the publications mentioned in footnote 1 of that article; Saliba, *A History*; Saliba, ‘Arabic Planetary Theories’, and Nikfahm-Khubravan & Ragep, ‘The Mercury Models’.

[45] O occurs as a point on the ecliptic meridian, as the beginning of the fourth sector of Mercury’s eccentric orb, and here as a point on the inner deferent. Although theoretically possible, it seems improbable that two of these occurrences indicate the same point, since the endpoint of the ecliptic meridian will usually be indicated on the parecliptic and the beginning of the fourth sector on the outer deferent. Similarly, the point D on the horizon will usually be indicated on the parecliptic and not on the inner deferent. In this case, a confusion with the letter *dhāl* may be possible, since that letter is not used for any other points.

L as their centre. The fixed point A is the apogee of the dirigent and T its perigee. Point H is the movable point on the deferent that is farthest removed from the centre L of the small circle KMS and hence indicates the direction of the deferent centre M as seen from L. Finally, point S is the centre of the *equant* (*muʿaddil al-masīr*), i.e., the centre of uniform motion of the epicycle centre.

5v: 25–29 Now we consider the epicycle $FQVU$ with centre Θ (the use of Q and Θ here is not in agreement with their previous and later occurrence). U is the body of the planet, F (on the extension of line $S\Theta$) the mean apogee of the epicycle, and Q (on the extension of line $E\Theta$) the true apogee. $\angle AS\Theta$ or $\angle ASF$, measured in counter-clockwise direction, is the mean centrum. Therefore $\angle TSF$ is the excess of the mean centrum over $180°$, as stated in the text. As was mentioned above, the deferent centre M has the same angular velocity on the small circle KMS as the centre of the epicycle Θ on the deferent, but in opposite direction. Since M is at point K when the epicycle centre is at the apogee A of the deferent, it follows that $\angle KLM$, measured in clockwise direction, is equal to $\angle ASF$, i.e., to the mean centrum. Thus $\angle TLH$ is the excess of the mean centrum over $180°$, and is hence equal to $\angle FST$.

5v: 29–3r: 3 The true centrum is $\angle AE\Theta$ or $\angle AEQ$, which differs from the mean centrum by $\angle E\Theta S$, the equation of centre (in the text: 'first equation'). Note that the true centrum is measured by the arc ABI on the parecliptic. The mean anomaly of the planet is $\angle F\Theta U$, and its true anomaly $\angle Q\Theta U$. Thus the difference between mean anomaly and true anomaly is $\angle F\Theta Q$, measured by arc FQ on the epicycle, which is equal to $\angle E\Theta S$, i.e., to the equation of centre. Then the 'corrected centrum' is the angle $\angle AEU$ under which the planet is seen from the Earth (measured by the arc $ABGIX$ on the parecliptic). It is obtained by adding the equation of anomaly ('second equation'), $\angle \Theta EU$, to the true centrum. The longitude or 'true position' (*taqwīm*) of Mercury, finally, is measured by the parecliptic arc between the vernal equinoctial point (not shown in the diagram and in Figure 9) and X. Note that the line EUX indicating the planetary longitude is clearly drawn in black in the diagram in the Horoscope.

3r: 3–6 The passage in the text concerning Mercury's direct and retrograde motion has already been discussed in Section 9, where we calculated that the planet will reach its first station when its true anomaly increases by $4\frac{1}{2}$ more minutes, rather than the text's 3 minutes. The statement about the ratio of $\frac{1}{2}UV$ to UE refers to the Theorem of Apollonius mentioned in Section 9. Here V is the point of intersection of the extension of line EU with the epicycle. For a simple epicycle model, Apollonius showed that the planet reaches its station when the ratio of $\frac{1}{2}UV$ to UE is equal to the ratio of the planet's mean motion in longitude to its mean motion in anomaly.[46] There is no reason to believe that ʿImād al-munajjim Maḥmūd al-Kāshī in fact calculated these ratios, since he could derive the necessary information from the table of planetary stations in any zīj.

The section on the configuration of the orbs of Mercury finishes by indicating the sectors of the eccentric orb and the epicycle of Mercury (cf. Section 10). The first and third sectors of the eccentric orb are said to start at points *A* and *T*, the apogee and perigee of the deferent. The second sector is said to start at a point Θ' (this cannot possibly be the same as the epicycle centre Θ) 'at the end of the golden line'. The fourth sector starts at O'' (certainly different from O and probably also from O', cf. footnote 45), which thus lies at the other end of the golden line. Unfortunately, the golden line is missing from the diagram in the manuscript or is invisible because the background of the diagram is also painted in gold. The line is in any case perpendicular to the apsidal line *AT*. Since the criterium used for the sectors is that 'according to motion', as is stated on fol. 3r: 11, the end points of the golden line signify the points where Mercury's equation of centre reaches its maximum value. These can be shown to correspond to values of the true centrum equal to approximately 92° and 268°. 3r: 6–13

The points where the equation of anomaly of Mercury reaches its maximum value can simply be determined by drawing tangents to the epicycle extending from the Earth *E*. Also these were supposed to have been drawn in gold in the diagram in the Horoscope, but are likewise missing or invisible. The first sector of the epicycle starts at the true apogee *Q*, the third sector at the true perigee *J*, and the second and fourth sectors at the two tangent points.

The reconstruction shows that the following elements are correctly drawn in the manuscript diagram (cf. Figure 8 on p. 36):

- the location of the horizon (*BD*, horizontal black line) and the ecliptic meridian (OQ', vertical black line);
- the inner parecliptic (small circle in black on red) and the outer parecliptic (outermost circle in red, with graduation);
- the inner and outer dirigent (two circles in black around the centre of the small circle in the middle of the diagram);
- the small circle carrying the centre of the deferent (in red);
- the apsidal line *AT* (in red, cutting the epicycle somewhat to the left of its centre; this is the only line extending through the upper half of the diagram);

[46] See, for instance, Pedersen, *A Survey*, pp. 331–338.

- the line *LH* indicating the direction of the deferent centre *M* as seen from the centre *L* of the small circle carrying *M* (in red, to the left of the apsidal line *AT* and likewise cutting the epicycle);
- the lines *S*Θ*F* and *E*Θ*Q*, from which respectively the mean anomaly and the true anomaly are measured (in red, practically coinciding within the epicycle);
- the line Θ*U* connecting the epicycle centre with the planet and thus indicating its mean anomaly and true anomaly (in red).

The inner deferent (in red) seems to have been incorrectly drawn in between the inner parecliptic and the inner dirigent (note that the centre of the red circle in between these two orbs is in fact a point on the small circle carrying the deferent centre). The outer deferent is missing completely. The epicycle has been drawn with an inner and an outer surface, although these are not mentioned in the text.

In spite of the few inaccuracies in the orbs, the diagram in the Horoscope turns out to be also numerically a highly accurate representation of the actual configuration of Mercury at the time of birth of Iskandar. For instance, the distance of the apogee *A* from the ascendant *B* is 58° in the diagram and close to 57° according to the numerical data in the text. The true centrum is drawn in the diagram as approximately $6^s\,7.4°$ and is equal to $6^s\,7;53°$ according to the *Īlkhānī Zīj*. The mean anomaly can be calculated as 145° and is close to 146° according to the diagram. Finally, the equation of anomaly is calculated in the text to be 18;43° and can be measured from the diagram as 19.5°. (As was explained above, I have chosen to reconstruct the diagram for a mean anomaly equal to $7^s\,0°$, since otherwise all lines through the centre of the epicycle Θ would practically coincide. Therefore the above-mentioned angles from the text are slightly different in Figure 9.)

13. The Ascendant

The *ascendant* or *horoscopus* (*ṭāliᶜ*, pl. *ṭawāliᶜ*), the rising point of the ecliptic at a given time, is of prime importance in astrology. It is the most significant indicator for making predictions about the character of a newborn child and events in his or her life, and it lies at the basis of systems for dividing the ecliptic into twelve astrological houses, which are used to make further types of predictions. Since the exact time of birth is rarely known and the degree of the ascendant changes very rapidly in the course of time, several special methods, called *namūdār*, were proposed for correcting the ascendant at the time of birth, which are here discussed in Section 14.

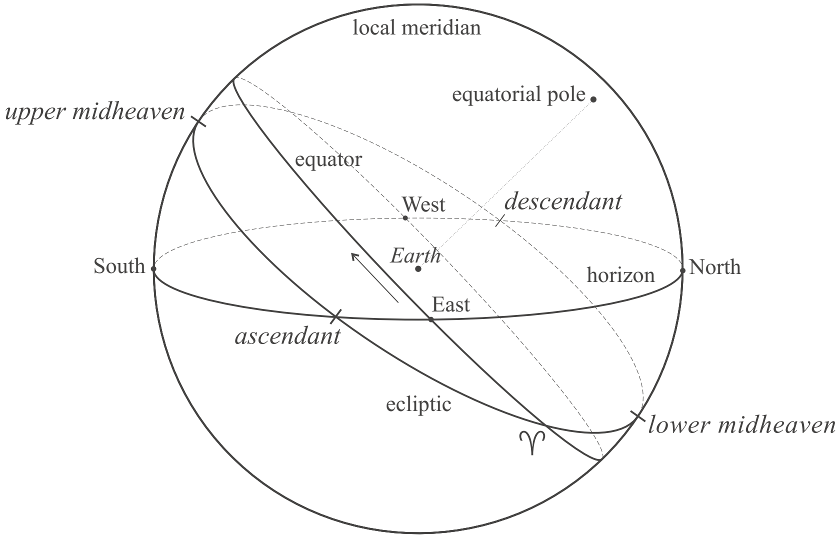

Figure 10: Definition of the four cardines: ascendant, descendant, upper midheaven, and lower midheaven. The arrow indicates the daily rotation of the celestial sphere.

At any given time, the ascendant is the eastern point of intersection of the horizon and the ecliptic (see Figure 10). Similarly, the *descendant*, the setting point of the ecliptic, is the western point of intersection of the horizon and the ecliptic. The point of the ecliptic that culminates at the given time, i.e., the southern point of intersection of the local meridian and the ecliptic, is *upper midheaven* ('midheaven', *wasaṭ al-samā*), while the northern point of intersection of the meridian and the ecliptic is *lower midheaven* (Arabic *rābiʿ* 'fourth [house]', Latin *imum coeli*). The four points thus defined are the *cardines* or 'cardinal points' (in Arabic *watad* 'pivot', pl. *awtād*). Note that, as the points of intersection of two great circles, the ascendant and the descendant, as well as upper and lower midheaven, are 180° removed from each other (cf. Section 6). On the other hand, the distances between the ascendant or descendant and upper or lower midheaven vary in the course of a day (being 90° when the equinoctial points cross the horizon).

Right and Oblique Ascensions

In order to calculate the position of the ascendant at a given time (as well as many other quantities in spherical astronomy), Greek and Islamic astronomers used two extremely convenient functions, namely the *oblique ascension* and the *right ascension* (the latter being a special case of the former). The oblique ascension projects a point on the ecliptic

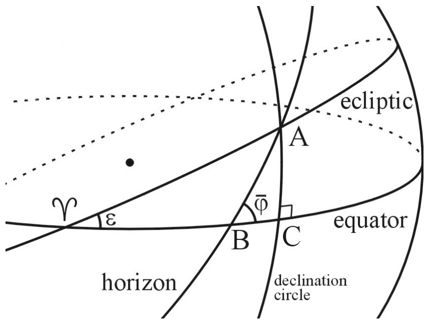

Figure 11: Right and oblique ascensions

(given by its longitude measured from the vernal equinox) onto a point of the equator (indicated by the equatorial arc between the vernal equinox and the projection) in such a way that both points rise simultaneously at the locality concerned. For example, in Figure 11 the ecliptic point *A* with longitude ♈*A* is projected on point *B* of the equator, which crosses the horizon at the same time; hence the oblique ascension of point *A* is the equatorial arc ♈*B*.

The oblique ascension depends on two parameters, namely the obliquity of the ecliptic ε and the geographical latitude of the locality φ. In the special case that the latitude is 0° (and hence the horizon is perpendicular to the equator), the oblique ascension is called right ascension and simply represents an orthogonal projection from the ecliptic onto the equator. In Figure 11 the horizon then coincides with the declination circle through *A*, the point of the equator rising simultaneously with *A* is *C*, and the right ascension is the equatorial arc ♈*C*.

In Arabic and Persian the oblique ascension is called *maṭāliʿ al-burūj bi balad* or *maṭāliʿ al-burūj bi ʿarḍ al-balad* ('ascensions of the zodiacal signs for the latitude of the locality') and the right ascension *maṭāliʿ al-burūj bi l-falak al-mustaqīm* ('ascensions of the zodiacal signs at sphaera recta'); both are occasionally abbreviated as *matāliʿ al-burūj* ('ascensions of the zodiacal signs') when it is clear from the context which ascension is meant.[47]

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[47] In this commentary, we do not need the actual methods for calculating the right and oblique ascensions. The right ascension $\alpha(\lambda)$ for a point with ecliptic longitude λ can be found from $\tan\alpha(\lambda) = \tan\lambda \cdot \cos\varepsilon$ or $\sin\alpha = \tan\delta(\lambda)/\tan\varepsilon$, where ε is the obliquity of the ecliptic and δ the declination (cf. Section 16). The oblique ascension is obtained as the sum or difference of the right ascension and the equation of daylight (cf. Section 18). See also Neugebauer, *HAMA*, Book I, Sections A3 and A4, pp. 30–45; Pedersen, *A Survey*, pp. 99–101 and 110–115; and Van Brummelen, *Mathematical Tables*, pp. 111–119.

By the above definitions, the oblique ascension maps the ascendant onto the rising point of the equator (i.e., the east point) and the descendant onto the setting point (the west point). These are both 90° removed from the meridian, and hence from the upper and lower culminating points of the equator. It follows that, by working on the equator, we can easily find the rising and setting points from the culminating points and vice versa, and, since the daily rotation of the celestial sphere takes place at a uniform pace around the axis of the equator, we can easily adjust for differences in time. By taking the inverse oblique ascension of the result (i.e., in medieval terms, by finding the argument in the oblique ascension table corresponding to the result), we can then simply find the corresponding points of the ecliptic.

In the Horoscope of Iskandar Sultan we find four applications of the technique sketched above.

1] *Determination of the ascendant at the time of the nativity.* We have seen that the time of birth of Iskandar Sultan was observed as 4 equal hours after sunset, i.e., shortly after 11 pm, on Sunday, 24 April 1384. For this time ʿImād al-munajjim Maḥmūd al-Kāshī finds the true solar longitude as $1^{s}\,12;38,45°$. If he now takes the oblique ascension of this longitude, he obtains the point of the equator that will rise simultaneously with the Sun in little less than six hours. However, because the time of the nativity is given with respect to sunset rather than sunrise, he uses the following trick. He considers the point precisely opposite the Sun, i.e., the point on the ecliptic with longitude $7^{s}\,12;38,45°$ (in the text this point is referred to as *naẓīr*). By means of the oblique ascension he finds the equatorial point that rises simultaneously with the point opposite the Sun, namely 235;57°. Since the point opposite the Sun rose precisely four equal hours ago, at the same time when the Sun set, it follows that the equatorial point that rises at the time of the nativity is $235;57° + 4 \cdot 15° = 295;57°$. By taking the inverse oblique ascension of this point, al-Kāshī finds the ascendant at the time of the nativity as $9^{s}\,0;56°$, or 0;56° Capricorn. 2v: 9–16

Next he calculates the longitude of upper midheaven, in the text indicated by ‘the tenth’ since it is the tenth of the twelve houses that divide the ecliptic (see Section 15). As was mentioned above, the culminating point of the equator can be found in a straightforward way because it is 90° removed from the rising point. Thus, at the time of the nativity, the culminating point of the equator was $295;57° - 90° = 205;57°$. Since this point is the orthogonal projection of upper midheaven onto the equator (note that the local meridian is perpendicular to the equator), it follows that the longitude of upper midheaven is the inverse right ascension of the culminating point of the equator and can thus be calculated as $6^s\,27°$. Al-Kāshī abbreviates this calculation by using the *normed right ascension*, i.e., the right ascension measured from Capricorn instead of from Aries. This function simply adds 90° to the 'ordinary' right ascension of any ecliptic arc, so that it yields, for example, the oblique ascension of the ascendant as a function of the longitude of upper midheaven, since the former is equal to the right ascension of the latter plus 90°. Thus the longitude of upper midheaven can be found from the oblique ascension of the ascendant by taking the inverse normed right ascension.

2v: 21–4r: 1 2] *Calculation of the ascendant from upper midheaven*. According to a method of correcting the ascendant attributed to Ptolemy, the degree of one of the cardines (disregarding its zodiacal sign) is equal to the degree of the planet that rules over the place of the conjunction or opposition of the Sun and the Moon preceding the nativity (for a more extensive discussion of this criterion and the concept of rulership, see 1] in Section 14 below). Since three days before the birth of Iskandar a Sun-Moon conjunction took place in 9;11° Taurus, whose ruler is Venus, ʿImād al-munajjim Maḥmūd al-Kāshī calculates the longitude of Venus at the time of the nativity and finds it to be $11^s\,27;17,56°$ (see the computational example on pp. 21–22). Since he had found upper midheaven (the tenth house) at the preliminary time of the nativity as $6^s\,27°$ (see 1] above), with the same number of degrees as the longitude of Venus, he concludes that the exact number of degrees of upper midheaven at the corrected time of the nativity must have been equal to that of Venus, and hence that its longitude was $6^s\,27;17,56°$. Now the culminating point of the equator can be found by taking the right ascension of this degree, namely 205;19,9°. The rising point of the equator is 90° removed from the culminating point and is hence 295;19,9°. Then the ascendant is found as above by taking the inverse oblique ascension, the result now being 0;26,49° Capricorn. The first two steps of this computation can be abbreviated by using a table for the normed right ascension, as in the previous example.[48]

[48] The computation in the Horoscope cannot be precisely reconstructed. An accurate calculation of the right ascension of upper midheaven yields 205;19,43°, and the use of al-Ṭūsī's right ascension table with values to seconds leads to 205;19,47°. Apparently for this step of the calculation a table with right ascension values to minutes (which would produce 205;19,6° if it were accurate) was used. An exact calculation of the inverse oblique ascension of 295;19,9° leads to an ascendant of 0;27,5° Capricorn, and the use of al-Ṭūsī's oblique ascension table for latitude 44° to 0;27,9° Capricorn. Also in this case the use of an oblique ascension table with values to minutes produces a result closer to the text, namely, 0;26,54° Capricorn.

3] *Determination of the time when the ascendant has a given value.* The method for correcting the ascendant that is attributed to Hermes requires that the ascendant at the time of the nativity is equal to the lunar longitude at the time of conception, and the other way around (for more details, see Section 14 below). In order to apply this criterion, first the day of conception is determined by means of a rough estimate. Then one needs to find the exact time on this day when the ascendant is equal to the lunar longitude at the preliminary time of birth. Al-Kāshī presents the intermediate steps of the calculation of this lunar longitude (fol. 4r: 9–14) and arrives at $2^{s}\,21;18,43°$. He then calculates the solar position at true midnight of the estimated day of conception, which he finds as $3^{s}\,27;24,45°$. Since the Sun reaches its point of lower culmination at true midnight, this is also the longitude of lower midheaven. By the method explained under 2] above we can now quickly find the ascendant at true midnight. However, since the ascendant itself is not needed here, the text only uses its oblique ascension, 29;28,18°. This we compare with the oblique ascension of the desired ascendant (namely, the lunar longitude at the preliminary time of birth), 56;4,16°. The difference, 26;35,58° or 1;46,23,52 hours, is the time from midnight till the moment when the ascendant is equal to the required lunar position (note that the longitude of the ascendant increases as a function of time), and hence also defines the exact time of conception. The method of Hermes proceeds by determining the lunar longitude at the time of conception, which then becomes the corrected ascendant at the time of the nativity. 4r: 2–26

4] *Determination of upper midheaven from the ascendant*. By the third and last method for correcting the ascendant, which is associated with Abū Maʿshar, the ascendant is found as the average of the corrected ascendants computed by the other two methods (see also Section 14 below). The procedure for finding upper midheaven from the ascendant is that indicated under 1]: the oblique ascension of the ascendant is the normed right ascension of upper midheaven. 4r: 27–3v: 3

14. Correction of the Ascendant

After a preliminary value for the ascendant at the time of the nativity has been found (fol. 2v: 15; see also 1] in Section 13 above), ʿImād al-munajjim Maḥmūd al-Kāshī proceeds to correct the ascendant by three different canonical methods (*namūdār*, pl. *namūdārāt*, earlier Persian spelling *namūdhār*, Latin *animodar*), all three associated with well-known authors. The *namūdār* of Ptolemy is contained in his *Tetrabiblos*. Together with the method attributed to Hermes, it is also found in the 2v: 17–20

Thamara ('Fruit', in Greek: *Karpos*, in Latin: *Centiloquium*), a collection of one-hundred astrological aphorisms that was wrongly attributed to Ptolemy and was often copied together with the *Tetrabiblos*.[49] The third *namūdār* used by the author of the Horoscope is associated with Abū Maʿshar. *Namūdār*s appeared in the works of Islamic astrologers from early on.[50] They were also described in Persian zījes from the thirteenth to fifteenth centuries, including the *Īlkhānī Zīj* by Naṣīr al-Dīn al-Ṭūsī and the *Khāqānī Zīj* by Ghiyāth al-Dīn Jamshīd al-Kāshī. In addition, these sources present a *namūdār* associated with Zarathustra.[51]

2v: 21–4r: 1 1] *The method of Ptolemy*. The first *namūdār* is associated with Ptolemy himself and is found in the *Tetrabiblos*.[52] It is also included in the *Thamara*, in an extremely short form, as aphorism 34 (version with 100 aphorisms) or 36 (version with 102 aphorisms)[53] and can be formulated as follows: 'At the exact time of birth, the degree of one of the cardines (disregarding its zodiacal sign) is equal to the degree of the ruler over

[49] The origin of the *Thamara* is unclear (cf. http://ptolemaeus.badw.de/work/190). All extant Greek versions and Latin translations may ultimately derive from the Arabic. Lemay, 'Origin and Success' suggested that the *Thamara* was created by Abū Jaʿfar Ibn al-Dāya, the author of the popular commentary on the work that was also translated into Latin multiple times (the Arabic version of this commentary was edited and translated into Italian in Martorello & Bezza, *Commento al Centiloquio*). However, there is no reliable evidence for this hypothesis and the contents of the 100 aphorisms rather points to a Greek origin. See also Sezgin, *Geschichte des Arabischen schrifttums*, vol. VII, pp. 42, 44–45 and 157, and Ullmann, *Die Natur- und Geheimwissenschaften*, pp. 283–284. A widespread Persian commentary was written by Naṣīr al-Dīn al-Ṭūsī around 1260 (edited in Zanjānī, *Šarḥ-e Samare*) and was later translated into Arabic. The Greek text was edited in Boer, *Καρπός*. Lemay died before he was able to publish his major study on the Arabic and Latin versions of the *Thamara*. Boudet has continued the Latin part of this work, summarised in Boudet, 'The Medieval Latin Versions'. A full edition and study of the most widespread Latin version, which was translated by Plato of Tivoli in 1136, is currently being prepared by Emanuele Rovati.

[50] See, for example, al-Qabīṣī (Burnett et al., *Al-Qabīṣī (Alcabitius)*, pp. 108–111), Kūshyār b. Labbān (Yano, *Kūšyār Ibn Labbān's Introduction*, ch. III.3, pp. 161–167), and al-Bīrūnī (Wright, *The Book of Instruction*, §§525–526, pp. 328–331).

[51] For al-Kāshī's extensive discussion of *namūdār*s in the *Khāqānī Zīj*, see Kennedy, 'Treatise V'. For the *namūdār*s, a direct dependence of the author of Iskandar's Horoscope on the *Īlkhānī Zīj* is less plausible, since the details of the descriptions of the methods of Ptolemy and Hermes in Section 1 of Treatise 4 of the zīj are different from those in the Horoscope, and al-Ṭūsī does not include the method of Abū Maʿshar.

[52] See Robbins, *Tetrabiblos*, ch. III.2, pp. 228–235, and the explanation in North, *Horoscopes and History*, pp. 51–52.

[53] See Martorello & Bezza, *Commento al Centiloquio*, pp. 114–117 (no. 34), and Zanjānī, *Šarḥ-e Samare*, pp. 38–39 (no. 36). The Greek version in Boer, *Καρπός*, p. 44 (no. 34) deviates significantly from the Arabic and does not refer to a nativity.

the ecliptic degree of the conjunction or opposition of the Sun and the Moon preceding the time of the nativity.'

Every degree of the ecliptic is assumed to be ruled, or dominated, by a planet (in Arabic and Persian called *mustawlī*). In Islamic sources this ruler is usually determined as the planet that has the largest sum of the weighted 'shares' over the five essential astrological dignities, namely, the domicile, exaltation, triplicity, term and decan.[54]

The Horoscope states (without providing a full calculation) that the conjunction of the Sun and the Moon preceding the birth of Iskandar was at 9;11° Taurus. Note that at the time of the nativity the Sun was in 12;38,45° Taurus (fol. 2v: 14) and the Moon in 21;18,43° Gemini (fol. 4r: 14). Thus the Moon had passed the Sun by approximately 38;40°. Since the daily lunar mean motion in longitude (approximately 13;10,35°) exceeds that of the Sun (0;59,8°) by 12;11,27°, it follows that a conjunction of the Sun and the Moon must have occurred approximately $\frac{38;40}{12;11,27} \approx 3$ days before the time of the nativity. Thus this conjunction would have taken place on 21 April 1384, at which time the Sun was indeed in 9° Taurus. An accurate calculation of the time of the conjunction should take into account that in particular the lunar equations change significantly during a period of three days. Computing with al-Ṭūsī's parameters (although not, in this case, with the actual tables from the *Īlkhānī Zīj*), I verified that at Uzgand the conjunction took place at 9:46 am on 21 April 1384 at a longitude of 9;11° Taurus, in complete agreement with the value given in the Horoscope.[55]

The author of the Horoscope states without explanation that 9° Taurus is ruled by Venus. Taurus is the domicile and the triplicity by day of Venus, so that of all planets Venus indeed has the largest sum of the 'shares' over the five astrological dignities. Al-Kāshī then proceeds to determine the longitude of Venus at the estimated time of the nativity as $11^{s}\,27;17,56°$. Since the position of upper midheaven had been estimated to be at 27° Libra (fol. 2v: 16; see 1] in Section 13), its degree in Libra is close to the degree of Venus in Pisces (disregarding the zodiacal sign), whereas the degree of the ascendant in Capricorn, 0;56°,

[54] The concept of rulership or domination and criteria for its determination are discussed, among others, in: Robbins, *Tetrabiblos*, I.20–22, pp. 90–111; Burnett et al., *Al-Qabīṣī (Alcabitius)*, pp. 58–61; Yano, *Kūšyār Ibn Labbān's Introduction*, I.22, pp. 66–69; Wright, *The Book of Instruction*, §§445–456, pp. 259–268, and Díaz-Fajardo, 'The Ptolemaic Concept'.

[55] With modern tables or software, in particular the program Alcyone Ephemerides by Rainer Lange and Noel Swerdlow, one finds that the conjunction took place at 10:04 am local time at a longitude of 9;14° Taurus.

is quite different.[56] Hence the degree of upper midheaven is assumed to be equal to that of Venus, so that we find the corrected position of upper midheaven as 27;17,56° Libra. By a standard application of the tables for oblique and right ascensions (see 2] in Section 13), al-Kāshī then finds the ascendant as 0;26,49° Capricorn.

4r: 2–26 2] *The Method of Hermes*. The second *namūdār* is included in the *Thamara* as aphorism 51 (version with 100 aphorisms) or 53 (version with 102 aphorisms)[57]: ‘The lunar longitude at the time of birth is equal to the ascendant at the time of conception, and the lunar longitude at the time of conception is equal to the ascendant at the time of birth.’

This method is applied by first approximating the time of conception from the time of birth by a numerical procedure that is also found in zījes.[58] The mean time of gestation (in Arabic and Persian called *makth*, i.e., ‘stay’ or ‘delay’) is taken to be equal to ten lunar cycles in longitude, i.e., since the daily lunar mean motion in longitude equals approximately 13;10,35°, to $\frac{10\cdot 360^\circ}{13;10,35} \approx 273;12,58^d \approx 273^d5^h11^m$ (text: $273^d5^h12^m$). The mean time of gestation is then corrected by the ‘equation of gestation’ (*taʿdīl-i makth*), which is found as the number of days that the Moon is removed from the ascendant at the time of birth under the assumption that it moves at the rate of its mean longitude. If the Moon is below the horizon, the equation of gestation must be added to the mean gestation, otherwise it must be subtracted. In the case of Iskandar’s birth, the lunar longitude was $2^s21;18,43^\circ$ and the preliminary ascendant 0;56° Capricorn. Thus the Moon was below the horizon and $5^s20;22^\circ$ removed from the ascendant. The equation of gestation, found as $5^s20;22^\circ/13;10,35 \approx 12;55,47^d \approx 12^d22^h19^m$, must therefore be added to the mean gestation, yielding a total period of gestation of $286^d3^h30^m$ (text: 286^d2^h).

[56] Al-Qabīṣī states explicitly that one should choose the cardine whose degree in its sign is *closer* to that of the ruling planet (Burnett et al., *Al-Qabīṣī (Alcabitius)*, pp. 110/1). Note that this implies that, as the result of an application of Ptolemy’s *namūdār*, the ascendant may change by as much as 29°, and therewith the time of birth by up to roughly two-and-a-half hours at the latitude of Uzgand.

[57] See Boer, *Καρπός*, pp. 48–49 (no. 51); Martorello & Bezza, *Commento al Centiloquio*, pp. 144–147 (no. 51), and Zanjānī, *Šarḥ-e Samare*, pp. 48–49 (no. 53). Only al-Ṭūsī’s commentary attributes this *namūdār* explicitly to Hermes.

[58] See, for instance, Vernet, ‘Un tractat d’obstetrícia’; King, ‘A Hellenistic Astrological Table’, esp. Section 7, pp. 699–701; Díaz-Fajardo, ‘Gestation Times’, and Chabás & Goldstein, *A Survey*, pp. 223–227. The rationale behind the *namūdār* of Hermes is discussed in Appendix D; see also Kennedy, ‘Treatise V’, pp. 140–143.

Calculating back 286^d4^h from the time of birth, we find a first approximation to the time of conception as 7 pm on Monday, July 13, 1383 (i.e., 5 Ābān 752 Yazdigird). The text omits the following step, namely the verification that the true lunar longitude at this time is in fact close to the preliminary ascendant at the time of birth.[59] It can be checked that, according to the *Īlkhānī Zīj*, the lunar longitude reached the required value in the early hours of 5 Ābān. The exact time of conception is then found from the condition that the ascendant at the time of conception is equal to the lunar longitude at the time of birth, $2^s\,21;18,43°$. ʿImād al-munajjim Maḥmūd al-Kāshī first calculates the solar position at midnight of Ābān 4 as $3^s\,27;24,45°$, from which he finds the oblique ascension of the ascendant at midnight as 29;18,18° (see 3] in Section 13). Since the oblique ascension of the required ascendant at the time of conception, $2^s\,21;18,43°$, is 56;4,16°, the conception must have occurred $(56;4,16° - 29;28,18°)/15° = 1;46,23,52$ hours after midnight. The text of the Horoscope next gives the number of hours from noon up to the 'corrected time of conception' as 13;35,10,52. The correction of −0;11,13 hours that has taken place here is for the equation of time and is in agreement with the table for that function found at the end of the third treatise of the *Īlkhānī Zīj*.[60] Finally, the lunar position at the corrected time of conception is calculated and the result, $9^s\,0;8,22°$, is taken as the corrected ascendant at the time of birth.[61]

3] *The Method of Abū Maʿshar*. A third *namūdār*, attributed to Abū Maʿshar, simply averages the results of the two previous methods. It is the result of this method that is applied in the further calculations in the Horoscope.[62] 4r: 27–4v: 3

[59] In Appendix D it is shown that the exact time of conception may differ by more than a day from the approximated time.

[60] For manuscript references to this table, see footnote 30. For a brief explanation of the equation of time, an overview of its uses in the Horoscope, and a discussion of the mistake in its application that was made in the calculation of the true solar longitude at midnight in the present context, see Appendix C.

[61] Only in this calculation is the addition of 30 minutes to the mean lunar longitude, as a correction to al-Ṭūsī's original mean motion parameters, explicitly mentioned (cf. Section 7.2 and footnote 28).

[62] Ghiyāth al-Dīn Jamshīd al-Kāshī mentions the possibility to average the results of the *namūdār*s of Ptolemy and Hermes, but does not associate this procedure with any author (cf. Kennedy, 'Treatise V', p. 143). The zīj of Ibn Maḥfūẓ al-Baghdādī associates the *namūdār* that averages the results of the *namūdār*s of Ptolemy and Hermes with Māshāʾallāh (Paris, Bibliothèque nationale de France, arabe 2486, fols 191v–192r; see https://gallica.bnf.fr/ark:/12148/btv1b100374746/f195.item). I have not found any sources besides the Horoscope of Iskandar that associate it with Abū Maʿshar.

15. Calculation of the Astrological Houses

4v: 4–29 The four cardines defined in Section 13 are the basis for a division of the ecliptic in twelve so-called ‘houses’ (Arabic: *bayt*, pl. *buyūt*; Persian: *khāna*), which are numbered in the order of increasing longitude. The ascendant is the first house, lower midheaven the fourth, the descendant the seventh, and upper midheaven the tenth. The intermediate ‘cusps’ (*marākiz*, lit. ‘centres’, the beginnings of the houses) can be defined in a number of different ways of varying mathematical complexity. The calculation of the houses is referred to in Arabic and Persian as *taswiya al-buyūt*, the ‘equalisation of the houses’.

In the Horoscope of Iskandar, the explanation of two common methods for the equalisation of the houses is announced. However, due to one or more missing folios in the manuscript, the end of the first method and the whole second method are missing from the text. The first method to be expounded is called the ‘famous method’; it is the standard method included in most Islamic sources and will be explained below. The sec-
4v: 6 ond method is called ‘verified cusps’ (*marākiz-i muḥaqqaqa*) in the text of the Horoscope and is referred to as ‘prime vertical method’ in the modern literature. This method divides the four quadrants of the prime vertical (the great circle passing through the zenith and the east and west points of the horizon, in the Horoscope called *dāʾira-yi awwal-i sumūt*, ‘circle of initial azimuth’) in three equal parts; the houses are then found as the points of intersection of the great circles through the trisection points on the prime vertical and the northern and southern points of the horizon with the ecliptic.[63]

The Standard Method

The standard method for determining the twelve houses can be described as follows. Assume that the ascendant and descendant, as well as upper and lower midheaven have been found as explained in Section 13 (cf. Figure 10). By definition, the ascendant is the beginning of the first house, lower midheaven of the fourth, the descendant of the seventh,

[63] This method is further discussed in North, *Horoscopes and History*, pp. 30–40, and Kennedy, ‘The Astrological Houses’, pp. 541–543. Also the two extant versions of the Horoscope for Iskandar’s half-brother Rustam (see footnote 23) contain general descriptions of the standard method and the prime vertical method with calculations according to both. For further Islamic methods of calculating the houses, see, for example, Kennedy, ‘Ibn Muʿādh’; Hogendijk, ‘Applied Mathematics’; Casulleras, ‘Mathematical Astrology’; Casulleras, ‘Métodos para determinar’, and Casulleras & Hogendijk, ‘Progressions, Rays and Houses’.

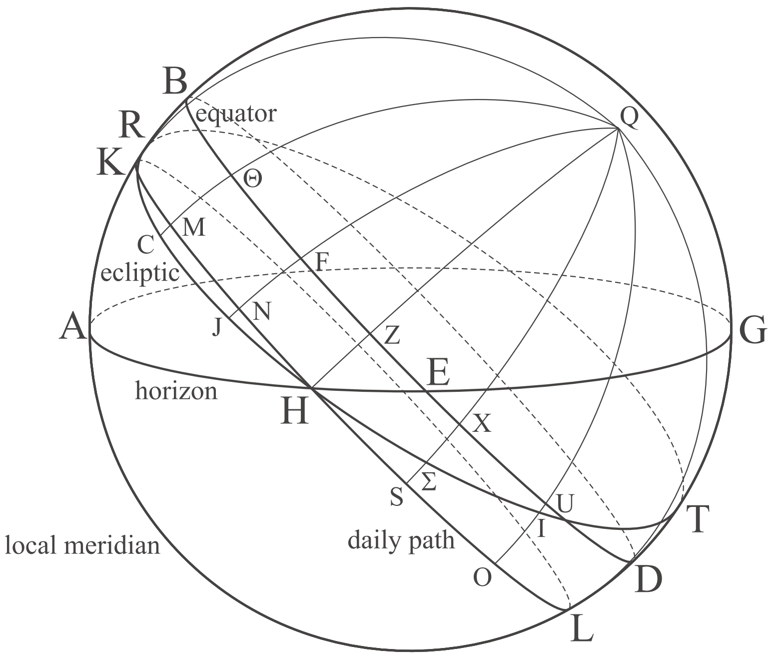

Figure 12: Equalisation of the houses according to the ‘Standard Method’

and upper midheaven of the tenth. Project the four cardines orthogonally (i.e., from the equatorial pole) onto the equator. Divide each of the resulting four arcs into three equal parts and project these back onto the ecliptic from the equatorial pole. The resulting twelve segments of the ecliptic are the twelve houses.

In the text of the Horoscope, the equalisation of the houses by the standard method is formulated in a slightly different way, illustrated by a defective diagram on fol. 4v, here reproduced in Figure 13 on p. 55. Figure 12 shows my own diagram for the actual situation at the time of Iskandar’s birth, with points on the sphere represented by the same letter symbols as in the Horoscope; some obvious differences from the diagram in the manuscript will be discussed below. The circle *ABGD* represents the local meridian at the time of the nativity, *AEG* the eastern half of the horizon, *BED* the eastern half of the equator, and *RHT* the eastern half of the ecliptic. Thus point *R* is upper midheaven, *H* the ascendant, and *T* lower midheaven. Now the half-circle *KHL*, drawn parallel to the equator, is the path that the ascendant traces on the celestial sphere during the day of Iskandar’s birth (note that *KHL* is generally *not* a great circle). *KH*, the half arc of daylight associated with the ascendant, is divided into three equal parts *KM*, *MN*, and *NH*, and similarly *HL*, 4v: 5–14 the half arc of nighttime, is divided into three equal parts HS, SO and OL.[64] The points K, M, N, H, S, O, and L are projected onto the equator and the ecliptic by intersecting 'declination circles' (*dawāʾir-i mayl*)[65] through these points with the equator and the ecliptic. The resulting points on the equator are B, Θ, F, Z, X, U, and D, and those on the ecliptic are R, C, J, H, Σ, I, and T.

The part of the text that states that C is the beginning of the eleventh house, J of the twelfth, Σ of the second, and I of the third, is missing from the manuscript. The same holds for the actual calculation of the longitudes of these cusps, which can be performed as follows. The arcs $B\Theta$, ΘF and FZ on the equator are equal, and each correspond to two seasonal day-hours associated with the ascendant. Similarly, the arcs ZX, XU and UD each correspond to two seasonal night-hours (note that a day-hour 4v: 14–15 and a night-hour add up to two equal hours). Furthermore, the given seven points on the equator are the endpoints of the ascensional arcs of the corresponding points on the ecliptic. This implies that the houses can be found as the inverse right ascensions of the seven points on the equator. In Section 14 we have found the ascendant according to the method of Abū Maʿshar as 0;17,35° Capricorn, the tenth house as 27;6,24° Libra, and the right ascension of the latter as 205;8,47° (measured from Aries). Since the right ascension of the ascendant is 270;19,10°, the half arc of daylight BZ or KH equals $270;19,10 - 205;8,47 = 65;10,23°$. Dividing this into three equal parts of 21;43,28°, we find the right ascension of the eleventh house as 226;52,15° and that of the twelfth house as 248;35,42°. By taking inverse right ascensions, we find that the eleventh house is in 19;20,9° Scorpio and the twelfth house in 10;13,39° Sagittarius. Since the half arc of daylight and the half arc of nighttime add up to 180°, it follows that ZD equals 114;49,37°. Thus the right ascensions of the second and third houses become 308;35,42° and 346;52,15° respectively, and their ecliptic longitudes 6;12,8° Aquarius and 15;43,48° Pisces. The remaining houses follow immediately, since the descendant is 180° removed from the ascendant and hence the equatorial arc between B and

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[64] The arc of daylight associated with an arbitrary point on the celestial sphere is defined as the part of the daily path of that point that lies above the horizon. Similarly, the arc of nighttime is the part of the daily path of a given point that lies below the horizon. Only for the Sun, these definitions correspond to the actual day and night. See further Section 18.

[65] The term *dāʾira-yi mayl*, here translated as 'declination circle', stands for a great circle through the equatorial poles, along which the declination (i.e., the distance from the equator) can be measured. Note that the modern term 'circle of declination' is also used for circles with a constant declination, which are parallel (rather than perpendicular) to the equator.

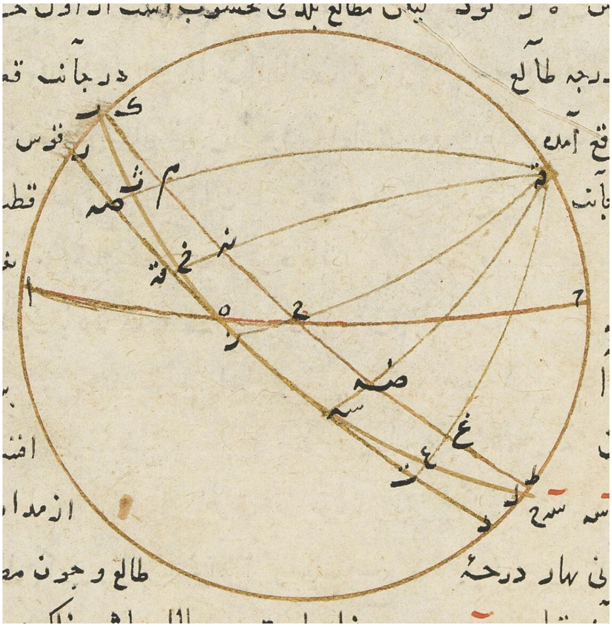

Figure 13: The calculation of the astrological houses as depicted in the manuscript of the Horoscope of Iskandar Sultan (London, Wellcome Library, Persian 474, fol. 4v).

the projection of the descendant onto the equator equals 180° minus arc *BZ*. As a result, the eleventh to third houses are precisely opposite the fifth to ninth houses. The longitudes found in this way are in agreement with the values to minutes given in the table on fol. 16v of the Horoscope of Iskandar Sultan. They are here reproduced in Appendix F.

The section on the equalisation of the houses mentions some properties that are not directly needed in the calculation of the houses. For instance, it is stated that arc *EZ* is the equation of daylight associated with 4v: 15–16
the ascendant, i.e., the difference between the half arc of daylight and 90°. *E* is the endpoint of the oblique ascension of the ascendant *H*, and *Z* the endpoint of its right ascension (said to be measured from Capricorn rather than from Aries). In the case of Iskandar's birth, the ascendant lies 4v: 18–23
south of the equator (and hence is not visible from the equatorial north pole) and the equation of daylight *EZ* must be subtracted from 90° in order to obtain the half arc of daylight. If the ascendant lies north of the equator, *EZ* must be added to 90° to obtain the half arc of daylight. It is not clear to me why, on lines 21 and 22, both in the case of a subtraction and in the case of an addition the half arc of daylight is indicated by 'excess' (*tafāḍul*). The last sentence on the page, which is not complete, 4v: 28–29
appears to indicate that the oblique ascension of the ascendant, which is

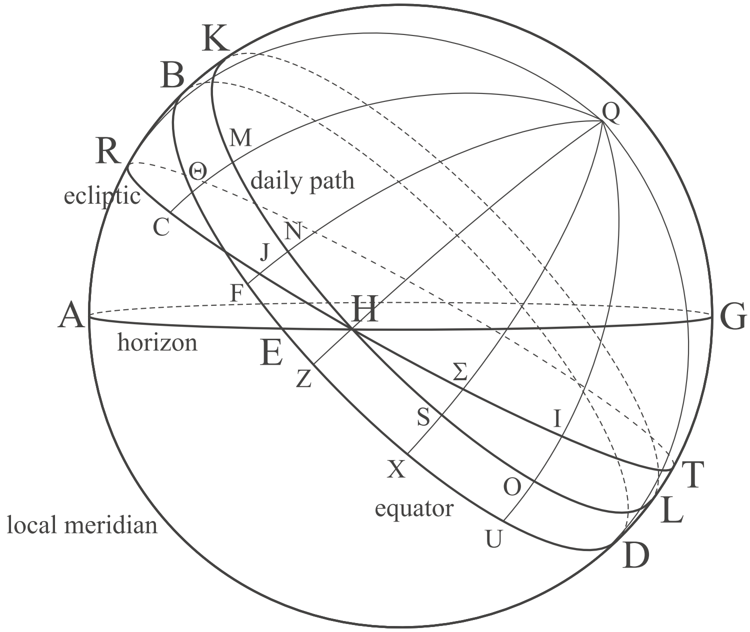

Figure 14: Equalisation of the houses as intended by the author of the Horoscope

measured from Aries, is 90° more than the right ascension of the tenth house measured from Capricorn (cf. Section 13). This right ascension was calculated on fol. 4v:3.

As far as the diagram on fol. 4v of the manuscript is concerned, it seems that some parts of the half-circles representing the equator, ecliptic and daily path were mixed up. We can note the following (cf. Figure 13):

- Points *B* and Θ of the equator lie above the ecliptic, the other points below.
- Of the points *K*, *M*, *N*, *H*, Σ, *I* and *T*, which, in the diagram, lie on a half-circle presumably supposed to represent the daily path of the ascendant, the first three in fact belong to the daily path, but the last three are points on the ecliptic and should lie above the daily path. The ascendant *H* lies both on the ecliptic and on the daily path.
- The points *S*, *O* and *L*, which lie on part of an arc between the ecliptic and the equator, belong to the daily path of the ascendant.

A reconstruction of the diagram that could have been intended by the author of the Horoscope is shown in Figure 14. Note that it preserves the relative positions of all letters indicated in the original diagram and that it restores the ecliptic and the daily circle by a minimal number of adjustments; only the depiction of the equator in the original diagram was basically correct. However, the reconstructed diagram is not in agreement with the text of the Horoscope to the extent that the ascendant *H* now lies above the equator instead of below and that the vernal equinoctial point lies between the ascendant and upper midheaven rather than between the ascendant and lower midheaven. This could only partially be remedied by assuming that *Q* stands for the southern instead of the northern pole, since in that case *R* would be the fourth house and *T* the tenth rather than the other way around. In any case, the diagram in the manuscript seems to be irreconcilable with the phrase in the text 'if the degree of the ascendant is hidden from the pole, as I depicted it on the diagram and as it happened in the case of the blessed ascendant, ...' (fol. 4v:20–21).

16. Distance of the Planets from the Equator

Now that the planetary longitudes and latitudes and the twelve astrological houses have been determined, ʿImād al-munajjim Maḥmūd al-Kāshī proceeds to make preparations for the calculation of further important indicators for the life of the native, namely planetary rays (explained in Section 21) and prorogations (cf. Appendix E). The following five sections in the Horoscope determine step by step the various spherical-astronomical quantities that are needed for this purpose. A central position in this process is occupied by the 'incidental horizon', also called 'position semicircle', the half-plane through the north and south points of the local horizon and the given heavenly body (for details, see Section 19). Various astrological quantities (besides the planetary rays and the prorogations also the astrological houses) are determined by dividing arcs on a great circle in equal parts or by setting off arcs of equal length on a great circle. There are several possibilities for choosing these great circles: besides the basic ones, i.e., the equator, the ecliptic and the horizon, other, more complicated possibilities are the prime vertical, seasonal hour lines, and the incidental horizon.[66] Al-Kāshī starts by explaining the calculation of basic concepts such as the distance from the equator of heavenly bodies not positioned on the ecliptic and the equation of daylight. After having given full details of the calculation of the incidental horizon, he introduces the 'corrected ascension' (Section 20), by means of which the planetary rays can be easily cast.

The distance of a heavenly body from the equator (i.e., in modern terms, its declination) is defined as the arc between the body and the equator on the declination circle passing through the body. As we have 3r: 14–16

[66] For a classification of such methods, see Casulleras & Hogendijk, 'Progressions, Rays and Houses' and the further literature mentioned in Section 21 and Appendix E.

seen above (cf. footnote 65), declination circles (*dawāʾir-i mayl*) are great circles through the equatorial poles perpendicular to the equator, as opposed to the modern 'circles of (constant) declination', which are parallel to the equator and are small circles except when the declination is zero. In the text the word *mayl* ('declination') is only used for heavenly bodies on the ecliptic, in particular for the Sun (fols 3r: 17, 7r: 14) and the lot of fortune (fol. 7v: 26; see Section 22). For bodies north or south of the ecliptic the term *buʿd ʿaz dāʾira-yi muʿaddil al-nahār* ('distance from the equator') is used exclusively.

As we have already seen in Section 5, the declination of the Sun can be calculated from its longitude by means of the formula $\sin\delta = \sin\varepsilon \cdot \sin\lambda$, where λ denotes the solar longitude, ε the obliquity of the ecliptic, and δ the solar declination. This formula follows from an application of the sine rule to triangle ΥQS in Figure 15, in which Υ is the vernal equinoctial point, Q the position of the Sun, and S the orthogonal
3r: 17–19 projection of Q onto the equator. The values given in the text for the sine of the declination and the declination itself leave no doubt that the obliquity used is 23;30°, the value consistently applied by al-Ṭūsī in the *Īlkhānī Zīj* and found in only very few other Islamic sources.

For an arbitrary heavenly body P not on the ecliptic, let Q be its orthogonal projection onto the ecliptic, R the point on the equator whose orthogonal projection onto the ecliptic is Q, and S the orthogonal projection of Q onto the equator (see Figure 15). Finally, let T be the orthogonal projection of P onto the equator. Arc ΥQ is the longitude λ of the heavenly body and arc QP its latitude β. The arc QS is the (first) declination δ of point Q, and arc QR is its second declination δ_2. The first declination can be calculated directly from the longitude λ according to the formula for the solar declination given above, while the second declination is found from $\tan\delta_2 = \sin\lambda \cdot \tan\varepsilon$ by means of an application of the 'double-tangent rule' to triangle ΥQR.[67]

[67] Another method for calculating the second declination that frequently occurs in medieval sources is expressed by the modern formula $\sin\delta_2 = \sin\delta(\lambda)/\cos\delta(90° - \lambda)$, which can be proved by the following application of the Rule of Four. Let X be the point on the ecliptic 90° removed from Q on the other side of the nearest equinox, and let Y be the orthogonal projection of X onto the equator. Let Z be the intersection of the declination circle through X and Y and the 'latitude circle' (i.e., the circle perpendicular to the ecliptic) through R and Q. Since X is a pole of this latitude circle, we have $XZ = 90°$ and hence $YZ = XZ - XY = 90° - \delta(90° - \lambda)$. The formula given above now follows from the Rule of Four applied to the triangles $\triangle RQS$ and $\triangle RZY$. The proof is given, for example, in *al-Zīj al-Jāmiʿ* of Kūshyār ibn Labbān; see Bagheri, *az-Zīj al-Jāmiʿ*, pp. 50 and 168–169 (English translation), p. 60 (commentary) and Arabic pp. 32 and 121–122.

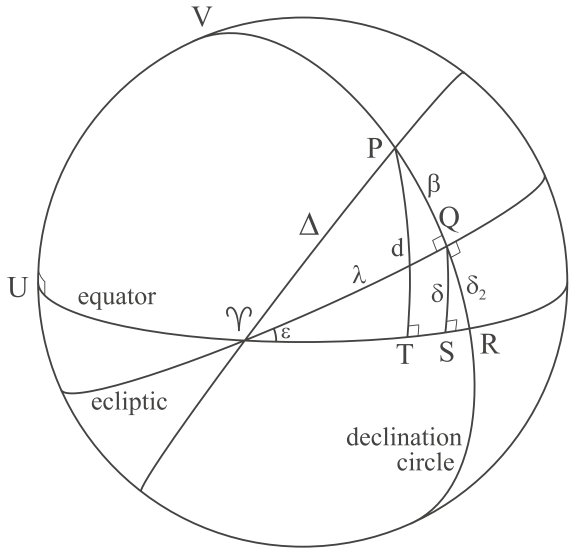

Figure 15: Calculation of the distance from the equator

The text of the Horoscope does not state in every detail how the distance from the equator $d = PT$ of the given heavenly body is calculated. In the six practical calculations for the Moon and the five planets, the following intermediate results are presented in each case: $\cos\beta$, $\cos\delta_2$, and $\sin PR$. In the text, $a_d = PR$ is called the 'argument of the distance' (*ḥiṣṣa-yi buʿd*); it is the sum of the absolute values of the latitude PQ and the second declination QR if P and R lie on different sides of the ecliptic, and their difference if P and R lie on the same side. As the last step in the calculation for each case $\sin d$ is found as a quotient, but it is not clear of which terms. Only for the Moon the 'sine of the distance from the equinox' (this distance is presumably $\Delta = \Upsilon P$) is mentioned, but its value has not been inserted.

Most probably, the author of the Horoscope found the distance from the equator by a method equivalent to the following formula, which is also found in other medieval sources:[68]

$$\sin d = \sin a_d \cdot \frac{\cos\varepsilon}{\cos\delta_2}.$$

This relation can be derived by noting that $\triangle TRP$ and $\triangle RUV$ (where U is the orthogonal projection of the solstitial point onto the equator,

[68] See, for example, Section II.5 in the *Zīj* of Ulugh Beg (Sédillot, *Prolégomènes publiés*, pp. 348–349, for the Persian text and Sédillot, *Prolégomènes traduction*, pp. 89–91, for a French translation).

which is 90 equatorial degrees removed from $\Aries$, and V is the pole of the ecliptic) are similar because they have the acute angle at R and a right angle in common. It follows from the Rule of Four that

$$\begin{aligned}\sin d = \sin PT = \sin PR \cdot \frac{\sin UV}{\sin VR} \\ = \sin a_d \cdot \frac{\sin(90^\circ - \varepsilon)}{\sin(90^\circ + \delta_2)} = \sin a_d \cdot \frac{\cos\varepsilon}{\cos\delta_2}.\end{aligned}$$

In this way $\cos\beta$ and the distance from the equinox Δ are not used (note that $\cos\beta$ is one of the terms from which Δ is calculated, since we have $\cos\Delta = \cos\lambda \cdot \cos\beta$ by the special case of the cosine rule, the 'Pythagorean formula for a spherical triangle', in triangle $\Aries QP$). Another straightforward method to calculate d is to first determine the angle at R in $\triangle QR\Aries$ by means of the sine law:

$$\sin\angle QR\Aries = \sin\lambda \cdot \frac{\sin\varepsilon}{\sin\delta_2},$$

and then to apply the sine law to $\triangle PRT$ to find

$$\sin d = \sin\angle QR\Aries \cdot \sin a_d = \frac{\sin\lambda \cdot \sin\varepsilon \cdot \sin a_d}{\sin\delta_2}.$$

Alternatively, the Rule of Four may be applied to the pair of triangles $\triangle QRS$ and $\triangle PRT$ to find $\sin d = \sin\delta \cdot \sin a_d / \sin\delta_2$. However, none of these methods gives a better correspondence to the intermediate results indicated in the text than the first method given above. A simple explanation for the inclusion of the values for $\cos\beta$ and $\cos\Delta$ in the calculations of the distance from the equator may be that they were going to be used in the determination of the ascension of transit in the following section of the Horoscope.

3r: 17–3v: 5 *Notes to the calculations in the text.* The given values of the second declination all confirm the use of al-Ṭūsī's obliquity value 23;30°. For the *Moon*, the quotient should be 0;19,0,58 instead of 0;19,4,58 (scribal mistake?). For *Mercury*, the argument of the distance is the sum of the second declination and the latitude (since the planet is north of the ecliptic and point R is south of it), i.e., 21;47,49°. The sine of this arc is 0;22,16,44, but the remainder of the calculation was performed with the value 0;22,10,44, apparently a scribal mistake.

A problematic case is *Jupiter*, for which three different declination values appear in the Horoscope. In the present calculation the arcsine of 20;27,6 should be 19;55,45° rather than the text's 19;15,45° (scribal error). In determining the ascension of transit (see Section 17) the complement of Jupiter's declination is

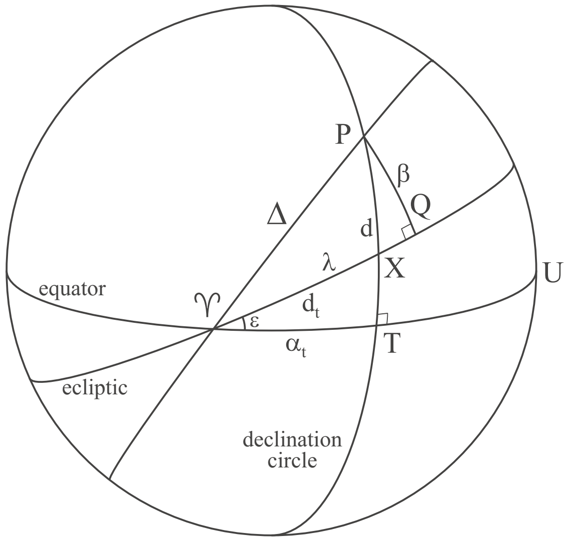

Figure 16: Calculation of the ascension of transit

given as 70;44,15°, which is in agreement with the erroneous value. Furthermore, the same erroneous value is repeated in the calculation of the latitude of the incidental horizon (see Section 19). However, from the calculation of the ascension of rising (see Section 18) the tangent of Jupiter's declination can be reconstructed as 19;47,58, which is the tangent of 18;15,45°, apparently an additional misreading of the above incorrect value 19;15,45. The same tangent value is used in Section 20 for finding the corrected ascension.

17. Ascension of Transit

A transit (*mamarr*) is defined as the crossing of any declination circle (*dāʾira-yi mayl*, i.e., a great circle through the poles of the equator, cf. footnote 65) by a given heavenly body. Thus, a transit in medieval astronomy is a more general concept than its modern meaning of the crossing of the local meridian (i.e., culmination). Let P be the heavenly body and T its orthogonal projection onto the equator (cf. Figure 16). Then the *ascension of transit* (*maṭāliʿ al-mamarr*) α_t is the equatorial arc between the vernal equinox ♈ and T (measured in the direction of the zodiacal signs). The longitude of the point of intersection X of the declination circle PXT with the ecliptic is called the *degree of transit* (*daraja-yi mamarr*) and is measured by the arc $d_t = \Upsilon X$. 3v: 6–10

The ascension of transit of a given heavenly body P can be calculated by considering the two right-angled triangles $\triangle \Upsilon PQ$ and $\triangle \Upsilon PT$ in Figure 16. Of the first triangle, only the side $\Delta = \Upsilon P$, the distance of the heavenly body from the vernal equinoctial point, is unknown, and it can be found by the Pythagorean formula for spherical triangles as

$$\cos\Delta = \cos \Upsilon Q \cdot \cos PQ = \cos\lambda \cdot \cos\beta.$$

In the second triangle we similarly have $\cos\Delta = \cos\Upsilon T \cdot \cos PT$, so that we find

$$\cos\alpha_t = \cos\Upsilon T = \frac{\cos\Delta}{\cos PT} = \cos\lambda \cdot \frac{\cos\beta}{\cos d}.$$

The calculations in the Horoscope proceed exactly by this method, except that the cosines are expressed in the text as the sines of the complementary arcs on the sphere.[69]

Once the ascension of transit is known, the degree of transit d_t can simply be found by an inverse lookup in a right ascension table, since any point on the declination circle through the heavenly body rises simultaneously with the heavenly body and T at *sphaera recta* (note that the declination circle is perpendicular to the celestial equator).

3v: 10–6r: 11 *Notes to the calculations in the text.* For all planets, *nuqṭa* (lit. 'point') stands for U, the projection of the solstitial point nearest to the heavenly body concerned onto the equator (thus U is 90 equatorial degrees removed from the vernal equinoctial point Υ). Since it is not clear how this is implied by the general term *nuqṭa*, it may be more appropriate to adopt the translation 'point [of ascension of the nearest solstice]' from the expression used in this context in the Horoscope of Rustam (cf. footnote 23 on p. 21).

The calculations for the Moon and the five planets are basically correct. Only for *Venus*, the quotient should be 0;59,57,1,57 rather than 0;59,57,55 (scribal or computational error?), but the following arcsine was undoubtedly calculated from the value given in the text. The degrees of transit were omitted for each planet. If calculated from the given ascensions of transit by means of al-Ṭūsī's obliquity value 23;30°, they become: Moon 2^s 21;36,48°; Saturn 2^s 15;36,13°; Jupiter 2^s 2;45,10°; Mars 7^s 1;39,50°; Venus 11^s 27;52,30°; Mercury 2^s 0;3,38°.

18. Equation of Daylight and Ascensions of Rising and Setting

6r: 12–21 The ascension of rising and the ascension of setting of a given heavenly body, as well as its equation of daylight, half arc of daylight, and arc of daylight are defined on fol. 6r. In Figure 17, let us consider the daily path across the celestial sphere of the heavenly body P. As noted before,

69 See also Section II.11 in the *Zīj* of Ulugh Beg (Sédillot, *Prolégomènes publiés*, pp. 359–360, for the Persian text and Sédillot, *Prolégomènes traduction*, pp. 103–104, for a French translation).

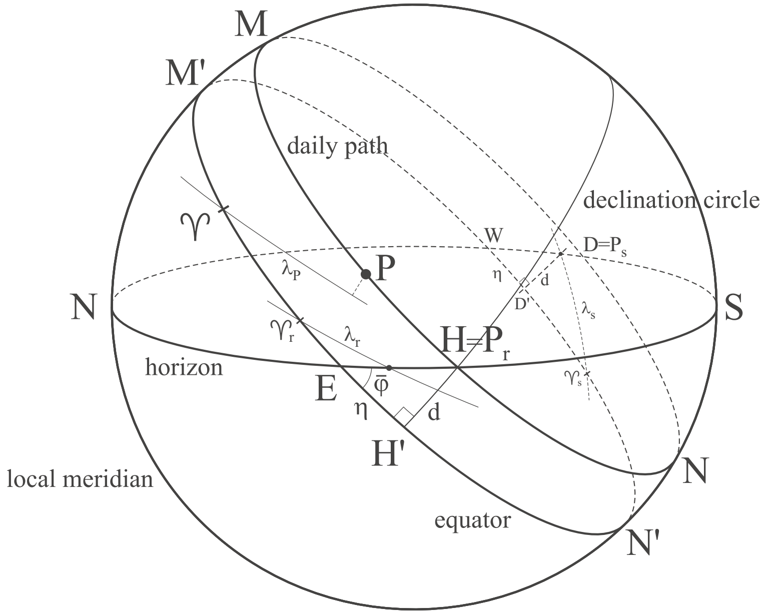

Figure 17: Calculation of the equation of daylight and the ascensions of rising and setting

this path is a small circle parallel to the equator, namely *NHPMD*, where *H* is the eastern intersection of the daily path with the horizon and *D* the western intersection. The 'arc of daylight' (*qaws al-nahār*) associated with the heavenly body is the part of the daily path that lies above the horizon (*HMD*), and the 'arc of nighttime' (*qaws al-layl*) is the part that lies below the horizon (*DNH*). Only in the case of the Sun do the arcs of daylight and nighttime correspond to the actual day and night. In all other cases they measure the time that the heavenly body concerned is above or below the horizon during the given day.

In order to calculate the arcs of daylight and of nighttime, we project the daily path of the heavenly body orthogonally onto the equator. *H* is the rising point of the daily path on the eastern horizon, *D* the setting point on the western horizon, *M* the point of upper culmination, and *N* the point of lower culmination. Then let H' be the orthogonal projection onto the equator of *H*, and M' the orthogonal projection of *M*. Let *E* be the east point of the horizon, i.e., the eastern intersection of the horizon and the equator, and ♈ the vernal equinoctial point. Note that M' is the culminating point of the equator, so that $EM' = 90°$.

Now the half arc of daylight (*niṣf-i qaws al-nahār*) is equal to the arc $M'H'$ on the equator, and the equation of daylight (*taʿdīl al-nahār*) is the difference $\eta = EH'$ of the half arc of daylight from EM', i.e., from 90°.

The equation of daylight can be calculated from the right-angled triangle $\triangle EHH'$ by using the 'double-tangent rule':

$$\sin\eta = \sin EH' = \tan HH' / \tan \angle H'EH = \tan d \cdot \tan\varphi.$$

Here d is the distance of the heavenly body from the equator calculated in Section 16, and φ the geographical latitude. In order to obtain the half arc of daylight, the equation of daylight must be added to $EM' = 90^\circ$ if the heavenly body is located to the north of the equator, and subtracted from 90° if it is to the south. The arc of daylight is twice the half arc of daylight.

The author of the Horoscope proceeds to calculate the degrees of rising and setting of each heavenly body, i.e., the degrees of the ecliptic that set and rise simultaneously with the body under the assumption that the body rises or sets at the time of the nativity. For this purpose, he introduces imaginary horizons that pass through the heavenly body but have the same latitude as the place of birth.[70] Alternatively, one may rotate the whole celestial sphere around the axis perpendicular to the equator until the heavenly body lies in the plane of the horizon; thus the heavenly body P will have moved to its rising point or its setting point. In Figure 17, $P_r = H$ denotes the position of the heavenly body P under the assumption that it rises at the time of the nativity, and $P_s = D$ its position under the assumption that it sets at the time of the nativity. Υ_r and Υ_s are the respective corresponding positions of the vernal equinox; in each of the hypothesized situations the ecliptic longitude of the heavenly body (λ_P, λ_r and λ_s in the figure) is supposed to be the same. The calculations are then carried out as follows:

The (oblique) ascension of the rising point $H = P_r$ of the heavenly body, i.e., the equatorial arc between the vernal equinox Υ_r and the Eastern point E, is found as the difference of the ascension of transit $\Upsilon_r H'$ and the equation of daylight EH' if the body is situated north of the equator, and as the sum of these two quantities if the body is south of the equator. Because of the symmetry of the celestial sphere with respect to the local meridian, the (oblique) ascension of the setting point $D = P_s$ of the heavenly body, i.e., the equatorial arc between the vernal equinox Υ_s and the Western point of the horizon W, is found as the sum of the ascension of transit $\Upsilon_s D'$ and the equation of daylight $D'W$ if the body

[70] Note the difference between these horizons and the incidental horizons discussed in Section 19 below. The latter pass through the given heavenly body as well as through the north and south points of the horizon, so that their latitudes are different from that of the local horizon except if the heavenly body is situated in the eastern half of the plane of the horizon.

is north of the equator and as the difference of these two quantities if it is south. However, the quantity that is usually called 'ascension of setting' (*maṭāliʿ-i ghurūb*) in Islamic sources is $\Upsilon_s W$ plus or minus 180°. When using this definition, the ascension of setting can also be found by adding the arc of daylight to the ascension of the rising point.[71]

Different from the Horoscope of Rustam, the one for Iskandar does not actually give the degrees of rising and setting of the Moon and the five planets. The degree of rising, i.e., the longitude λ_r of the point on the ecliptic that rises simultaneously with the body under the assumption that it rises at the time of the nativity, is the degree whose oblique ascension is equal to the ascension of rising. The degree of setting λ_s cannot be found directly from the oblique ascension table with the ascension of the setting point $\Upsilon_s W$, since we are here dealing with a setting phenomenon. We thus need to add 180° to $\Upsilon_s W$, thereby obtaining the 'ascension of setting', find the arc whose oblique ascension is equal to the value found, and then add 180° again to obtain λ_s. For example, the ascension of transit of the Moon is 80;52,2°, and its equation of daylight 18;49,34°. Since the Moon is north of the equator, the ascension of the setting point is found as 80;52,2° + 18;49,34° = 99;41,36°. The ascension of setting is equal to this value plus 180°, i.e., it is 279;41,36°, which is the oblique ascension of 256;49,47°. Thus we find the degree of setting as 256;49,47 − 180° = 76;49,47. Note that for heavenly bodies on the ecliptic the degrees of rising and of setting are equal to the ecliptic longitude (and, consequently, for bodies near the ecliptic the degrees of rising and of setting and the ecliptic longitude will be very close to each other).

Notes to the calculations in the text. The calculation of the equation of daylight of the Moon and the five planets is one of only two types of computations in which the division by 60 ('lowering', cf. Section 5), needed due to the use of trigonometric functions with a base radius of 60, is explicitly mentioned. For the *Moon*, the ascension of the rising point should be 62;2,38° instead of 62;2,28°. For *Saturn*, the given ascension of the rising point is one degree too high; this could have resulted from misreading the ascension of transit as 75;21,46° or from a computational mistake. For *Jupiter*, the product of the tangent of the distance from the equator and the tangent of the latitude should have been close to 0;21,0,28; it was apparently calculated for a misread value of the distance from the equator, 18;15,45° (instead of 19;55,45°, in the text: 19;15,45°; cf. Section 16, p. 60). The ascension of the rising point for *Mars* is based on the incorrect value 19;29,31° given earlier in the text; the correct 6r: 22–6v: 8

[71] cf. Section II.12 in the *Zīj* of Ulugh Beg (Sédillot, *Prolégomènes publiés*, pp. 361–362, for the Persian text and Sédillot, *Prolégomènes traduction*, pp. 105–106, for a French translation).

value is 209;29,31 + 11;13 = 220;42,31°. For *Mercury*, the product of the two tangents should have been near 0;22,31,46, but apparently the equation of daylight was calculated by taking the arcsine of a number close to 0;22,32,46. In each of the six cases the space for the ascension of setting is left blank. According to my calculations, the missing values should be as follows: Moon 279;41,36°, Saturn 276;47,42°, Jupiter 259;15,58° (from the incorrect equation of daylight; correct: 261;2,55°), Mars 18;16,31°, Venus 176;40,49°, and Mercury 259;56,31°.

Note that the products of the tangents of the distance from the equator and the geographical latitude are given to four sexagesimal digits for Saturn, Mars and Venus. In each case it can be verified that the multiplicands must have had precisely three digits. In particular, the value that was used for the tangent of the latitude of Uzgand, 44°, was 0;57,56,29 (exact value: 0;57,56,28,46,31,...).

19. Incidental Horizon

6v: 9–14 The *incidental horizon* (*ufuq-i ḥādith*) of a given heavenly body is defined in the Horoscope as the great circle passing through that body and the north and south points of the local horizon (see the shaded half-plane in Figure 18, in which *R* is the heavenly body, *A* the south point and *G* the north point of the local horizon). As explained on p. 57, ʿImād al-munajjim Maḥmūd al-Kāshī uses the incidental horizon as the reference plane for calculating the planetary aspects (see Section 21) and presumably also for finding prorogations (cf. Appendix E). He calculates the latitude of the incidental horizon for the Sun, the Moon, the five planets, and the lot of fortune.

Similarly to the local horizon, the latitude of the incidental horizon is defined as the arc (taken to be smaller than 90°) between the incidental horizon and one of the equatorial poles, measured perpendicularly to the incidental horizon, i.e., along the great circle $KN'ML$ through the poles of the equator and the poles of the incidental horizon. Thus, in Figure 18, the latitude of the incidental horizon is the arc $\varphi_i = ML$.

If the heavenly body happens to be rising or setting, the local horizon and the incidental horizon lie in the same plane. If the heavenly body happens to be in upper or lower culmination (i.e., if it crosses the local meridian), its incidental horizon coincides with the local meridian and has zero latitude. If the planet or star is in the eastern half of the celestial sphere, the orientation of the incidental horizon is taken to be the same as that of the local horizon; if the planet or star is in the western half of the celestial sphere, the orientation of the incidental horizon is taken to be opposite to that of the local horizon (i.e., for an observer in the northern hemisphere, it is assumed to be ‘negative’ or southerly).

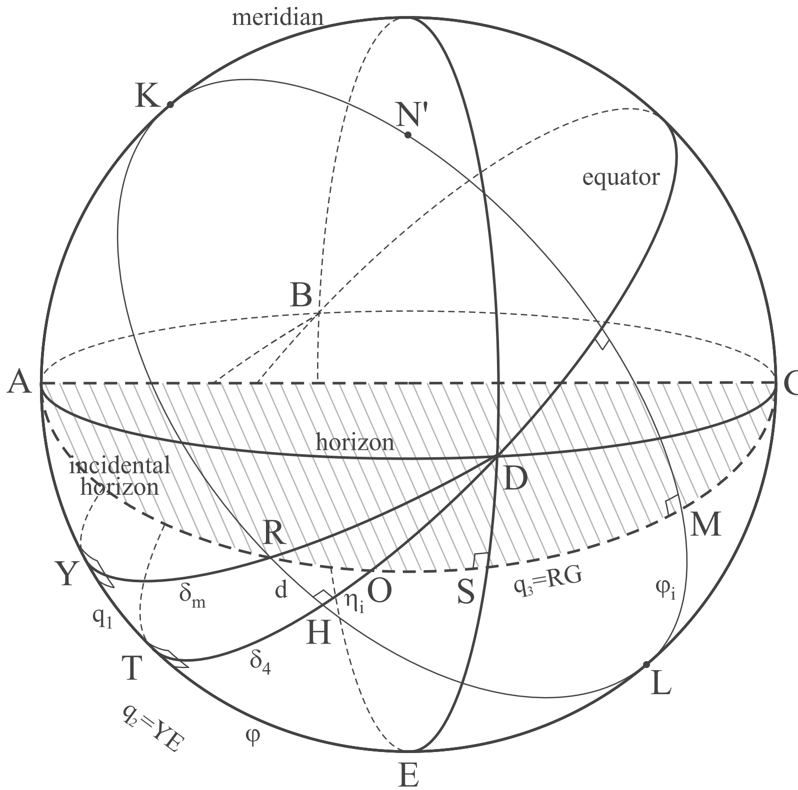

Figure 18: Calculation of the incidental horizon and the incidental equation of daylight

The method of calculating the latitude of the incidental horizon is shown step by step in the Horoscope. In the diagram provided for this purpose on folio 6v in the manuscript (see Figure 19), the view is onto the horizon from the nadir of the locality of nativity, and most arcs and points shown lie below the horizon (note that the proof is intended to be for the Sun, which was indeed below the horizon at the time of the nativity). In my own Figure 18, the view is onto the local meridian from the west, showing the heavenly body *R* between its setting and its lower culmination. The letters from the diagram in the text have been maintained in all cases in order to facilitate a comparison of the two figures. 6v: 15–7r: 13

The calculation proceeds as follows: Let *ABGD* be the local horizon, with *A* the north point, *B* east, *G* south, and *D* west. Let *DTB* be the equator, with *T* its point of lower culmination, and let *K* and *L* be the equatorial north and south poles. Then *AELGK*, with *E* the nadir of the geographical locality for which the calculations are made, is the

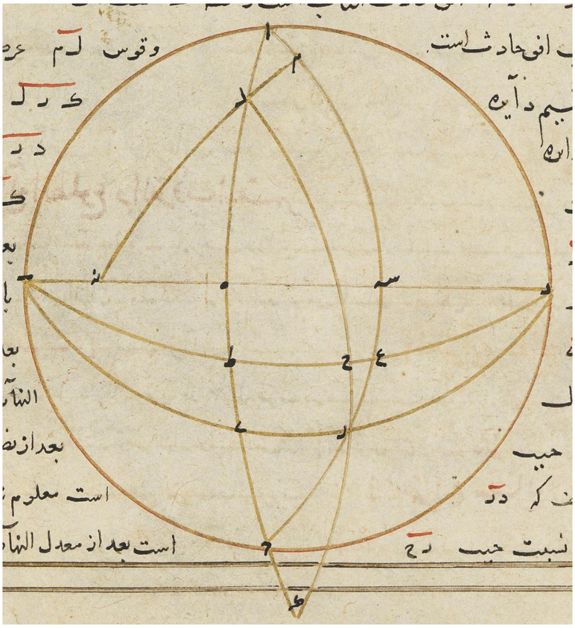

Figure 19: The calculation of the incidental horizon and the incidental equation of daylight as depicted in the manuscript of the Horoscope of Iskandar Sultan (London, Wellcome Library, Persian 474, fol. 6v).

local meridian, and BED is the prime vertical (in the Horoscope called 'circle of initial azimuth', *dāʾira-yi awwal-i sumūt*). Let R be the given heavenly body (in the text assumed to be the Sun) and N' the pole of its incidental horizon ARG (in text and diagram in the Horoscope reference is made to the opposite pole N). Finally, let M be the intersection of the great circle $KN'L$ through the poles of the equator and the incidental horizon with the incidental horizon. Then the latitude of the incidental horizon is $\varphi_i = \text{arc}\, LM$, measured along $KN'L$. Now φ_i can be calculated in the following five steps.

1] Let KRL be the declination circle of the heavenly body R, which crosses the equator orthogonally in H. Then $d = RH$ is the distance of the heavenly body from the equator (cf. Section 16). Let DRB be the great circle passing through the heavenly body and the west and east points of the horizon; this great circle intersects the meridian perpendicularly in Y. Now the two right-angled triangles KHT and KRY have the acute angle at K in common. It therefore follows from the Rule of Four that

$$\frac{\sin HT}{\sin HK} = \frac{\sin RY}{\sin RK}.$$

If we denote HT (the distance in right ascension between the heavenly body and lower midheaven or the fourth house[72]) by δ_4 and RY (the distance of the heavenly body from the meridian) by δ_m, we thus find $\sin\delta_m = \sin\delta_4 \cdot \cos d$.[73] Since δ_4 can be calculated as the difference of the ascension of transit α_t and the right ascension of lower midheaven α_4, which are both known, we can now determine δ_m, and therewith its complement $DR = 90° - \delta_m$, the distance of the heavenly body from the west point D.

2] Now the two right-angled triangles $\triangle DRH$ and $\triangle DYT$ have the acute angle at D in common. Thus we find from the Rule of Four that

$$\frac{\sin TY}{\sin HR} = \frac{\sin YD}{\sin RD},$$

so that the so-called 'first arc' $q_1 \stackrel{\text{def}}{=} TY$ can be obtained from $\sin q_1 = \sin d / \cos\delta_m$.

3] Since $\varphi = ET$ is the latitude of the locality of nativity, the 'second arc' $q_2 = EY$ can simply be found by adding the latitude to the first arc: $q_2 = q_1 + \varphi$.[74]

4] Now let S be the point of intersection of the incidental horizon with the prime vertical, so that RS is the distance of the heavenly body from the prime vertical. The two right-angled triangles $\triangle DRS$ and $\triangle DEY$ have the acute angle $\angle RDS = \angle YDE$ in common. Thus we find from the Rule of Four that

$$\frac{\sin RS}{\sin YE} = \frac{\sin RD}{\sin ED} \qquad \text{or} \qquad \frac{\sin RS}{\sin q_2} = \cos\delta_m,$$

from which the 'third angle' $q_3 = GR = 90° + RS$ follows immediately.

5] Finally note that the triangles $\triangle GRY$ and $\triangle ALM$ both have a right angle (respectively at Y and M), and also have an acute angle in common, since the respective angles at G and A are equal to the angle between the

[72] If the heavenly body is situated above the plane of the local horizon, the distance in right ascension between the body and upper midheaven (the tenth house) will be taken here. As in most other calculations, such alternative cases are left to the reader.

[73] This step is one of only two places in the Horoscope in which the division by 60 ('lowering', cf. Section 5), needed due to the use of trigonometric functions with a base radius of 60, is explicitly mentioned, both in the proof and in the actual calculations for the Moon and Saturn.

[74] In case the heavenly body lies south of the equator and under the local horizon, q_2 is found as the *difference* of the first arc and the geographical latitude. Other cases, for instance if the heavenly body is above the horizon, are left to the reader.

incidental horizon and the local meridian. Thus we find from the Rule of Four that

$$\frac{\sin GR}{\sin AL} = \frac{\sin RY}{\sin LM} \qquad \text{or} \qquad \frac{\sin q_3}{\sin \varphi} = \frac{\sin \delta_m}{\sin \varphi_i}.$$

It follows that the latitude of the incidental horizon can be found from

$$\sin \varphi_i = \sin \delta_m \cdot \frac{\sin \varphi}{\sin q_3}.$$

Note that the statement that, in the case of the Sun and the diagram in the text, the latitude of the incidental horizon is southerly because its pole N lies south of the equator is correct. This condition is in fact equivalent to the one given earlier, namely that the latitude of the incidental horizon is southerly when the star is west of the local meridian.

To summarize the above procedure, we find consecutively:

- δ_m, the distance of the star from the meridian, from
  $\sin \delta_m = \sin(\alpha_t - \alpha_4) \cdot \cos d$;
- the first auxiliary arc q_1 from $\sin q_1 = \sin d / \cos \delta_m$;
- the second auxiliary arc q_2 from $q_2 = q_1 + \varphi$;
- the third auxiliary arc q_3 from $q_3 = 90° + \arcsin(\sin q_2 \cdot \cos \delta_m)$;
- and the latitude of the incidental horizon φ_i from
  $\sin \varphi_i = \sin \delta_m \cdot \sin \varphi / \sin q_3$.

Two somewhat different methods for calculating the latitude of the incidental horizon can be found in the *Zīj* of Ulugh Beg.[75]

7r:14–7v:29 *Notes to the calculations in the text.* For each of the planets as well as the lot of fortune, the distance from the fourth house is calculated by subtracting a constant value for the ascension of the fourth house, namely 25;8,47°, from the ascension of transit. Calculating with al-Ṭūsī's value for the obliquity of the ecliptic, 23;30°, we find that this value stems from an ecliptic longitude of the fourth house close to 27;6,25° Aries. Consequently, the underlying longitude of the tenth house is close to 27;6,25° Libra, which differs by only a second from the result of Abū Maʿshar's method for correcting the ascendant and upper midheaven (see Section 14).

For the *Sun*, the sine of the declination (i.e., the distance from the equator) is mentioned, whereas the sine of its complement, 0;57,46,10, is needed for the calculation. The given 'sine of the first arc' is in fact the sine of the complement of δ_m, but for all other planets the first arc is correctly taken equal to YT.

[75] See Section IV.1.2 in the *Zīj* of Ulugh Beg (Sédillot, *Prolégomènes publiés*, pp. 438–440, for the Persian text and Sédillot, *Prolégomènes traduction*, pp. 205–208, for a French translation).

For the *Moon*, the sine of the distance from the meridian was found by multiplying the given sine of the distance from the cardine by the value 0;56,54,15 instead of the correct value 0;56,54,25 for the cosine of the distance from the equator. For *Saturn*, the second arc should be 75;10,16° rather than 75;11,1°, since the arcsine of the correctly computed quotient 0;31,3,21 is 31;11,1°. For *Jupiter*, the distance from the equator should have been 19;55,45°, but the calculations were made for the value 19;15,45° given in the text (cf. p. 60). Because *Venus* lies between the equator and the prime vertical at the time of the nativity, its second arc should have been obtained by subtracting the first arc from the geographical latitude instead of adding the two. For *Mercury*, the sine of the distance from the circle of initial azimuth should have been 0;48,19,18,56 (seconds and thirds interchanged); this is in better agreement with the multiplicands that constitute this product ($\sin q_2$ and $\cos \delta_m$) as well as with the following sine. The *lot of fortune*, whose longitude is here given for the first time, was calculated from the longitude 0;17,35° Capricorn for the ascendant as $270;17,35 + 42;38,40 - 81;18,43 = 231;37,32°$ (cf. Section 22), i.e., from the value found by means of Abū Maʿshar's *namūdār*. Using the given distance from the cardine, we can compute the ascension of transit of the lot of fortune as 229;10,56°, which is not an accurate value for the right ascension of its longitude. The given declination, however, is clearly based on al-Ṭūsī's value for the obliquity of the ecliptic. Various steps have been left out: the distance of the lot from the meridian is 22;45,41° and its second auxiliary arc 66;40,22°.

20. Corrected Ascension and Corrected Degree

After having found the latitude of the incidental horizon, ʿImād al- 8r: 1–3
munajjim Maḥmūd al-Kāshī reaches the endpoint of his series of calculations by determining the 'corrected ascension' (*maṭāliʿ-i muṣaḥḥaḥ(a)*) of each heavenly body, i.e., the distance from the vernal equinox of the point of intersection of the incidental horizon with the equator, point O in Figure 18. This point will serve as the reference point for the calculation of planetary rays in Section 21 and also appears to have been used as the starting point of the prorogations in the tables on fols 23v–62v of the Horoscope (cf. Appendix E).

The corrected ascension is found as the difference or the sum of the ascension of transit (the distance from the vernal equinoctial point of the orthogonal projection H of the planet onto the equator, cf. Section 17) and the 'incidental equation of daylight' $\eta_i = HO$ (*taʿdīl al-nahār-i ḥādith*). ʿImād states incorrectly that the incidental equation of daylight can be found from

$$\frac{1}{\tan LM} = \frac{\sin RH}{\tan HO}, \qquad \text{i.e.,} \qquad \frac{1}{\tan \varphi_i} = \frac{\sin d}{\tan \eta_i},$$

and suggests that this follows from an application of the Rule of Four to the pair of triangles $\triangle ROH$ and $\triangle RLM$ by mentioning that '*RL* (sic!) is the maximum sign, *LM* is the local latitude (sic!), and *RO* is the distance of the planet from the equator'. The correct formula for η_i, which is in fact applied by ʿImād al-munajjim in his calculations, is $\sin\eta_i = \tan\varphi_i \cdot \tan d$. This follows most easily from the similarity of the incidental equation of daylight to the ordinary equation of daylight, the only difference being that the place of the local horizon is now taken by the incidental horizon, so that the angle $\angle HOR$ between the equator and the incidental horizon is equal to the complement of the latitude of the incidental horizon φ_i.

Similar to the ordinary equation of daylight, in order to obtain the corrected ascension the incidental equation of daylight must be subtracted from the ascension of transit if the heavenly body is north of the equator, and added to it if the body is south of the equator. The corrected degree (*daraja-yi muṣaḥḥaḥ(a)*) of the heavenly body can then be found as the arc whose oblique ascension for the latitude of the incidental horizon is equal to the corrected ascension. Since most Persian zījes from the Timurid period contain oblique ascension tables only for a range of integer northern latitudes, this implies that interpolation between values from two different tables or even a separate calculation (if the incidental latitude lies outside of the tabulated range) might be necessary. If the latitude of the incidental horizon is southerly, the corrected degree can be found from an oblique ascension table for the corresponding northern latitude, namely by finding the inverse ascension of the corrected ascension plus or minus 180°; the corrected degree is then the ecliptic degree 180° removed from the result.

8r: 4–8v: 3 *Notes to the calculations in the text.* For the *Moon*, the product of the two tangents should have been 0;18,35,23 rather than 0;18,36,23. For *Jupiter*, the distance from the equator is here taken as 18;15,45°; the incidental equation of daylight should have been 14;28,14° instead of 14;29,14°. For the *lot of fortune*, the tangent of the latitude of the incidental horizon should have been 0;32,42,41 rather than 0;31,42,41. The corrected degrees on the ecliptic are never mentioned. Using an exact formula for the inverse oblique ascension, we find the following values: Sun 1^s 12;38,40°, Moon 2^s 17;2,29°, Saturn 2^s 14;30,27°, Jupiter 2^s 0;57,46°, Mars 7^s 1;35,52°, Venus 11^s 28;12,20°, Mercury 2^s 0;47,44°, lot of fortune 7^s 21;20,50°. That the corrected degree of the lot of fortune is not equal to its ecliptic longitude is due to inaccuracies in the calculation of its incidental horizon. The corrected degree for Jupiter differs too much from its longitude, which is undoubtedly caused by the two incorrect values for its distance from the equator that occur in the Horoscope (cf. the 'Notes to the calculations' in Sections 16 and 18).

21. Projection of the Rays

According to the doctrine of the ‘projection of the rays’ (*maṭraḥ-i shuʿāʿ* or *maṭāriḥ-i ashiʿʿa*), the Sun, the Moon and the planets cast seven rays of astrological significance to particular points of the ecliptic, the ‘aspects’ (*naẓar*, pl. *anẓār*). By the simplest definition, these points are 60, 90, 120, and 180 ecliptic degrees removed from the longitude of the planet. They are respectively called the ‘sextile’ (*tasdīs*), ‘quartile’ (*tarbīʿ*), ‘trine’ (*tathlīth*), and ‘opposition’ (*muqābala*), where the rays emanating from the planet in the direction of increasing longitude determine the ‘left aspects’ and those in the opposite direction the ‘right aspects’. However, in practice most medieval astrologers preferred more complicated methods of calculating the aspects involving right or oblique ascensions.[76] 8v: 4–9

ʿImād al-munajjim Maḥmūd al-Kāshī casts the planetary rays by using, for each given planet, the intersection of its incidental horizon with the equator as the reference point, as was also done by various Maghribian astronomers and by Ulugh Beg. Thus he adds arcs of 60°, 90° and 120° to, and subtracts them from, the corrected ascension of each planet (see Section 20) and then takes the inverse oblique ascension for the latitude of the incidental horizon to obtain the aspects. As a result, the aspects cross the incidental horizon precisely two, three or four hours before or after the planet.

Notes to the calculations in the text. I have recomputed all aspects given in the Horoscope and found that they generally contain errors of 10 minutes at most (the opposite aspect is given for none of the planets). The errors tend to become larger if the arguments of the inverse oblique ascension are further away from 0°, which suggests the use of slightly different latitude values. Since zījes usually contain oblique ascension tables only for integer degrees of latitude, it is plausible that the latitudes were rounded, or that some non-trivial type of linear interpolation was applied between tables for different latitudes. However, the errors are not systematic enough to allow conclusions about the exact method of calculation. Only for Mercury, the aspects in the Horoscope differ from recomputed values by up to four degrees, again for unclear reasons. 8v: 10–9r: 15

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[76] Various methods for the projection of the rays are described in detail in the following publications: Kennedy & Krikorian Preisler, ‘The Astrological Doctrine’; Hogendijk, ‘The Mathematical Structure’; Casulleras, ‘Ibn Muʿādh on the Astrological Rays’; Hogendijk, ‘Applied Mathematics’; Casulleras, ‘Ibn ʿAzzūz al-Qusanṭīnī’s Tables’; Casulleras, ‘El cálculo de aspectos’, and Casulleras & Hogendijk, ‘Progressions, Rays and Houses’. Ulugh Beg’s treatment of the planetary rays is found in Section IV.1.4 of his *Zīj*; see Sédillot, *Prolégomènes publiés*, pp. 441–442, for the Persian text and Sédillot, *Prolégomènes traduction*, pp. 209–210, for a French translation.

22. Astrological Lots

9r: 16–27 A separate section of the Horoscope on fol. 9r gives the positions of twenty-six astrological lots (*sahm*, pl. *sihām*) at the time of the nativity according to the doctrine of Abū Maʿshar al-Balkhī.[77] In the table on fols 16v–17r, which supplements the ecliptic longitudes of the houses, the planets, the nodes and the astrological lots at the time of birth with astrological information, the number of lots is increased to 45. All lots are found by means of simple arithmetical calculations from the longitudes of the Sun, Moon, and planets, and in each case the result is reckoned from the longitude of the ascendant. For many of the lots the calculation differs slightly depending on whether the time of the nativity is during the day or during the night. For example, during the day the longitude of the lot of fortune (*sahm al-saʿāda*) is found as the elongation of the Moon from the Sun, and at night as the elongation of the Sun from the Moon, in each case reckoned from the longitude of the ascendant. Since Iskandar was born at nighttime, the lot of fortune at the time of his nativity is found as $270;17,35 + (42;38,40 - 81;18,43) = 231;37,32°$, i.e., $21°38'$ Scorpio. The lot of courage (*sahm al-shajāʿa*) is found as the elongation of the lot of fortune from Mars during the day and as the elongation of Mars from the lot of fortune at nighttime, again reckoned from the longitude of the ascendant. Thus, for Iskandar's birth we find the lot of courage as $270;17,35 + (211;25,24 - 231;37,32) = 250;5,27°$, i.e., $10°5'$ Sagittarius.

23. Fixed Stars

9v–15r For 83 fixed stars from Ptolemy's star catalogue in Books VII and VIII of the *Almagest*, the Horoscope lists the ecliptic longitude, northern or southern latitude, and magnitude, and then calculates the second declination, 'argument of the distance', and distance from the equator (cf. Section 16); the ascension of transit and degree of transit (Section 17); and the equation of daylight, ascension of the rising point and degree of the rising point (Section 18). However, not all of these quantities are actually provided for each star. From the intermediate results that are occasionally given, we can see that the calculations proceed entirely according to the methods explained above for the Sun, Moon and planets.

[77] See, for instance, Burnett et al., *The Abbreviation*, Chapter 6 of the Arabic text, pp. 70–79. Even more extensive collections of lots can be found in Lemay, *Kitāb al-madkhal al-kabīr*, vol. III, pp. 613–660 (*Qawl* VIII), and Wright, *The Book of Instruction*, §§ 475–476, pp. 279–289. See also Haddad et al., 'Al-Bīrunī's Treatise'.

I will first list some possibly significant deviations in the longitudes, latitudes and magnitudes from the main traditions in the transmission of Ptolemy's star catalogue. In Band 3 of Kunitzsch, *Der Sternkatalog*, the variants in all available editions of the original Greek text and the Arabic and Latin translations are listed, supplemented with variants from various unedited manuscripts. Within the Arabic tradition, Kunitzsch distinguishes between the earlier translation of al-Ḥajjāj and that of Isḥāq ibn Ḥunayn corrected by Thābit ibn Qurra. Furthermore, he includes the coordinates from seven manuscripts of Naṣīr al-Dīn al-Ṭūsī's *Taḥrīr al-Majisṭī* and gives a concordance of the coordinates in three manuscripts of al-Ṣūfī's *Ṣuwar al-kawākib al-thābita* in comparison with those in the edition of that work in Schjellerup, *Description des étoiles fixes*. In deciding on the correct coordinates, Kunitzsch also made use of the critical analysis by the 12th-century scholar Ibn al-Ṣalāḥ of the differences in the star catalogues from five versions of the *Almagest* available to him.[78]

The longitudes in the Horoscope differ by precisely 18;56° from the *Almagest*. Elwell-Sutton has summarized the various rates of precession that could have been used to arrive at the values of the Horoscope over the period of time between the *Almagest* and the nativity of Iskandar (or the time of compilation of the Horoscope, 1411 CE) and, since he did not find a perfect agreement, has speculated about possible intermediate sources.[79] The rate of precession of one degree in seventy 9v: 2
years mentioned in the introduction of the section on fixed stars in the Horoscope is that of al-Ṭūsī in the *Īlkhānī Zīj*. It was also used by some later astronomers such as Ibn al-Shāṭir and Ulugh Beg. From further comparisons of the star names and the coordinates with, for instance, al-Ṣūfī, al-Ṭūsī's *Taḥrīr*, and al-Ṭūsī's Persian recension of the *Ṣuwar al-kawākib al-thābita* it may be possible to determine the precise source that was used for the information on the fixed stars in the Horoscope of Iskandar Sultan.

Deviations in the coordinates of the fixed stars in the Horoscope from those in the *Almagest* traditions with a longitude correction for precession of +18;56° are the following:

Butting One (*al-nāṭiḥ*, Aries #1e): Except for one manuscript, the Greek 10r: 13–16
sources contain the less accurate value 10;30° for the latitude of this star instead of 10;0° as found in the Horoscope (in the text the number of minutes is crossed out, but in the table on fol. 17v the zero minutes are unambiguous).

[78] This work was edited, translated into German and commented upon in Kunitzsch, *Zur Kritik der Koordinatenüberlieferung*.

[79] See Elwell-Sutton, 'A Royal Tīmūrid Nativity Book', pp. 126–127.

10r: 25–27 *Knee of the Enthroned Woman* (*rukba dhāt al-kursī*, Cassiopeia #5): The longitude differs from the main Greek and Arabic traditions of the *Almagest* by 18;36° instead of 18;56°, but it is in agreement with two manuscripts of al-Ḥajjāj's translation and the Latin tradition.

11r: 11–15 *Head of the Tyrant* (*raʾs al-jabbār*, Orion #1): The Greek tradition has latitudes −13;30° and −16;30° for this star instead of the common Arabic −13;50°. Some manuscripts of al-Ḥajjāj's translation and the Latin tradition have the Arabic scribal mistake −18;50° for −13;50°.

11v: 11–14 *Head of the Foremost Twin* (*raʾs al-tawʾam al-muqaddam*, Gemini #1): Most of the Greek sources give the latitude as 9;30° instead of 9;40°.

11v: 22–27 *Manger* (*al-miʿlaf*, Cancer #1): Most of the Greek sources give the latitude as 0;20° instead of 0;40°.

12r: 3–4 *Brighter of the Two Calves* (*anwar al-farqadayn*, Ursa Minor #6): Some of the Greek manuscripts give the longitude as 17;30° Cancer, but all other sources have 17;10° Cancer, which differs by precisely 18;56° from the longitude in the Horoscope.

12r: 22–25 *Further One of the Two Calves* (*al-tālī min al-farqadayn*, Ursa Minor #7): This star is of the third magnitude according to the Horoscope and the *Ṣuwar al-kawākib al-thābita* by al-Ṣūfī, but of the second magnitude in the *Almagest* traditions. Two manuscripts of al-Ḥajjāj's translation give the latitude as 74;30°.

12r: 26–27 *Lone Star in the Serpent* (*fard al-shujāʿ*, Hydra #12): In the *Almagest* traditions this star is of the second magnitude instead of the sixth.

13v: 8–11 *Forehead of the Scorpion* (*jabhat al-ʿaqrab*, Scorpius #6): In the *Almagest* traditions this star is of the fourth magnitude instead of the third.

13v: 11–14 *Leg of the Centaur* (*rijl Qanṭūris* (sic), Centaurus #35): Most of the Greek manuscripts and the Latin translation from the Greek made at Sicily have latitude −44;10° instead of −41;10° (scribal confusion of A and Δ).

14r: 1–4 *Head of the Dragon* (*raʾs al-tinnīn*, Draco #5): In the *Almagest* tradition this star is of the third magnitude instead of the first.

14r: 9–12 *Point of the Arrow* (*zujj al-nushshā[ba]*[80], Sagittarius #1): Some Greek manuscripts and a manuscript of the Arabic translation by Isḥāq ibn Ḥunayn give the latitude as −6;30° instead of −6;20°.

[80] This star is indicated as *zujj al-nshā* in the manuscript of the Horoscope. In both surviving Arabic translations of the *Almagest* its Greek name was translated as *naṣl al-sahm* (Kunitzsch, *Der Sternkatalog*, pp. 250–251). The name *zujj al-sahm* is found in al-Battāni's *Sābiʾ Zīj* and in Kūshyār b. Labbān's *Jāmiʿ Zīj*, which suggests that the use of the word *zujj* for 'arrowhead' stems from the early *Almagest* translation made for the caliph al-Maʾmūn, which is now lost (cf. Kunitzsch, *Der Almagest*, p. 294, no. 393, and Kunitzsch, *Zur Kritik der Koordinatenüberlieferung*, Anhang II, pp. 97–108). The form in the Horoscope is most probably a scribal mistake for *zujj al-nushshāba*. Sonja Brentjes kindly verified in her large collection of manuscript copies of al-Ṣūfī's *Ṣuwar al-kawākib al-thābita* that this name only appears in Paris, Bibliothèque nationale de France, arabe 5036 (and hence in the Hyderabad edition of al-Ṣūfī's work).

Beak of the Fowl (*minqār al-dajāja*, Cygnus #1): Some Greek manuscripts give the latitude as 49;0° instead of 49;20°. 14v: 1–4

Raised Part of the Kid's Tail (*shawla dhanab al-jadī*, Capricornus #28): The complete *Almagest* tradition except one manuscript of al-Ḥajjāj's translation give the latitude as northerly instead of southerly. 14v: 9–12

Mouth of the Horse (*fam al-faras*, Pegasus #17): Various Greek manuscripts as well as the Latin translation made from the Greek have latitude 2;30° instead of 22;30°. 14v: 13–16

Mouth of the [Southern] Fish (*fam al-ḥūt*, Aquarius #42/Piscis Austrinus #1): The *Almagest* gives the latitude as −20;20°, but one Greek manuscript and the Arabic and Latin traditions contain the erroneous value −23;0°. 14v: 17–20

Knee of the Fowl (*rukbat al-dajāja*, Cygnus #17): Some of the Greek manuscripts give the latitude as 64;45° instead of the common 63;45°. 14v: 25–27

Computational example. The complete calculations for the Left Leg of Orion (β Orionis) with longitude $\lambda = 2^{s}\,8;46^\circ$ and latitude $\beta = 31;30^\circ$ south would be as follows. Note that not all of the given intermediate steps are included in the text of the Horoscope for each star. 10v: 21–24

First the second declination is found as 22;3,45° as explained in Section 16. Since the Left Leg of Orion lies south of the equator and its projection onto the ecliptic north of it, the 'argument of the distance' is obtained by subtracting the second declination from the latitude, the result being 9;26,15°. Now the distance from the equator is found to be equal to 9;20,16° in southern direction (see again Section 16). The ascension of transit is calculated to be 71;46° as explained in Section 17. The degree of transit is then calculated by taking the inverse right ascension of the ascension of transit, yielding $2^{s}\,13;11^\circ$.

The tangent of the distance from the equator is found to an accuracy of two sexagesimal digits as 9;52. It is multiplied by the tangent of the latitude of Uzgand, 57;56, and the product divided by 60, namely 9;31,36,32, is the sine of the equation of daylight (cf. Section 18). By finding the arc corresponding to this value by means of linear interpolation in the sine table, namely between the values $\sin 9^\circ = 9;23,10$ and $\sin 10^\circ = 10;25,8$, the equation of daylight is obtained as 9;8,11°. Since the star lies south of the equator, the ascension of the rising point is now found as the sum of the ascension of transit and the equation of daylight (cf. Section 18), the result being 80;54°. By finding the inverse oblique ascension for the latitude of Uzgand from the obtained value we calculate the degree of the rising point to be $3^{s}\,13;40^\circ$. The ascension of setting, which is not given in the text, is 242;38°, and the degree of setting $1^{s}\,17;45^\circ$.

Notes to the calculations in the text. The calculations are generally accurate although incidentally small computational errors and reading mistakes appear to have been made by the author of the Horoscope. In many cases these errors propagate through the following steps of the calculations, so that they cannot be due to a careless copyist. It can be verified that the value for the tangent of the latitude of Uzgand used for the determination of the equation of daylight for all stars is equal to 57;56 (a more precise value for the latitude of 44° is 57;56,28,46,31, whereas we have seen on p. 66 that the value 57;56,29 was used for the calculation of the equation of daylight for three of the five planets). Also the tangent of the distance from the equator, which must be multiplied by the tangent of the latitude, was calculated to two places only and sometimes contains errors of 3 or 4 in the final digit. The product of these two tangents needs to be 'lowered' (i.e., divided by 60) in order to obtain the sine of the equation of daylight; this lowering is never explicitly mentioned in the text. In the translation as well as in the notes below I consistently refer to the lowered product and give the sine of the equation of daylight to base 60. The calculation of the arcsine of the product, which produces the equation of daylight, is usually given to seconds, i.e., more precise than the accuracy of the tangents allows. Small errors are often made in taking the inverse oblique ascension of the ascension of the rising point in order to obtain the degree of the rising point. Here the text values are in many cases up to ten minutes too large, possibly because of the use of (linear) interpolation in the oblique ascension table.

Detailed corrections:

9v: 14–17 *Wing of the Horse* (*janāḥ al-faras*): The second declination should be 0;28,42°, the argument of the distance 12;58,42°, and the distance from the equator 11;53,9°. However, these relatively small errors do not have any influence on the calculation of the ascension of transit and the equation of daylight.

9v: 21–23 *End of the River* (*ākhir al-nahr*): This star is south of the ecliptic instead of north. If its latitude were north, the star would indeed be circumpolar, but with the southern latitude its equation of daylight becomes 57;55,50°, the ascension of the rising point 99;32°, and the degree of the rising point 3^s 28;34°.

9v: 24–27 *Flank of the Chained Woman* (*janb al-musalsala*): The ascension of transit should be 9;41°, but the ascension of the rising point is in agreement with the incorrect ascension of transit. The sine of the equation of daylight appears to have been calculated as the product of the values 39;3 for the tangent of the distance from the equator (text: 39;0) and 57;56 for the tangent of the latitude (as usual), since only for these values is the arcsine of the lowered product, 37;42,18, equal to 38;56° when rounded to minutes.

10r: 1–4 *Foremost of the Two Signs* (*muqaddam al-sharṭayn*): The lowered product of the two tangents should be 17;24,44, whereas the sine of the given equation of daylight is 15;27,2. The correct product would have yielded: equation of daylight 16;52,46°, ascension of the rising point 4;3°, and degree of the rising point 7;36°.

10r: 25–27 *Knee of the Enthroned Woman* (*rukba dhāt al-kursī*): The degree of transit should be 0^s 13°.

10v: 1–4 *Wrist of the Pleiades* (*miʿṣam al-thurayyā*): The degree of transit and the tangent of the distance from the equator seem to have been copied from the second-next entry for the Middle of the Pleiades. The table on fol. 17v gives the correct ascension of transit, 24;12°, but the following degree of transit, $1^{s}\,20^{\circ}$, was calculated from the incorrect value in the text. Since the distance of this star from the equator can be calculated as 54;19°, the tangent of that distance should be close to 83;32, which indeed makes the star circumpolar.

Head of the Demon (*raʾs al-ghūl*): This is one of a number of cases where 10v: 5–8
the underlying tangent of the distance from the equator can be derived from the given sine of the equation of daylight, since we have $49;7 \cdot 57;56/60 = 47;25,29,32$. The correct value of the tangent is 49;9. The degree of the rising point is accurately calculated to minutes from the given ascension of rising of 345;53° as $11^{s}\,4;11^{\circ}$ (rather than the text's $11^{s}\,3;34^{\circ}$).

Cepheus (*fīqāwus*): The degree of transit is $11^{s}\,19;31^{\circ}$. 10v: 12–14

Eye of the Bull (*ʿayn al-thawr*): The arcsine of the given lowered product of 10v: 16–20
the tangent of the distance from the equator and the tangent of the latitude is in fact 15;28,41° (scribal mistake), but the ascension of the rising point was calculated with the equation of daylight given in the text, 15;23,41°.

[Left] Leg of Orion (rijl al-jawzāʾ): The calculated sine of the equation of day- 10v: 21–24
light is completely wrong and should be close to 9;31,39. The equation of daylight, however, seems to have been calculated as the arcsine of 0;8,30,54. The value for the equation of daylight given in the text was used for calculating the ascension of the rising point. Correct values would have been: equation of daylight 9;8,12°, ascension of the rising point 80;54°, degree of the rising point $3^{s}\,13;40^{\circ}$.

Forepart of the Belt (*muqaddam al-mintaqa*): The missing degree of transit 11r: 4–7
should be $2^{s}\,16;49^{\circ}$.

Head of the Tyrant (*raʾs al-jabbār*): The product of the given tangent of the 11r: 11–15
distance from the equator (9;29,0) and the tangent of the latitude of Uzgand (57;56), divided by 60, should be 9;9,24 instead of 9;8,24 (computational error?).

Hindpart of the Belt (*muʾakhkhar al-mintaqa*): The equation of daylight is com- 11r: 19–23
pletely wrong; it may have been found as the arcsine of 2;13,11 (reading mistake for the correct lowered product 2;43,11 given in the text?). Correct values are: equation of daylight 2;35,51°, ascension of the rising point 80;59°, degree of the rising point $3^{s}\,13;44^{\circ}$.

Little Goat (*judayy*): Accurate values for the ascension of transit and the degree 11r: 23–25
of transit are 1;29° and 1;37° respectively.

Shoulder of the Rein-Holder (*mankib dhī al-ʿinān*): The sine of the equation 11v: 1–4
of daylight should be close to 54;23,35, and hence the equation of daylight to 65;1,59°, the ascension of rising to 14;20°, and the degree of rising to $0^{s}\,26;11^{\circ}$.

Southern (Yemeni) Dog Star (*Shiʿrā al-yamāniyya*): The product of the two tan- 11v: 7–10
gents, divided by 60, should have been 16;22,56 instead of 16;17,56 (computational mistake?). The correct equation of daylight is 15;50,46°, the ascension of rising 111;10°, and the degree of rising $4^{s}\,7;31^{\circ}$.

11v: 11–14 *Head of the Foremost Twin* (*raʾs al-tawʾam al-muqaddam*): This star is *not* circumpolar. Its equation of daylight is 38;3,41°, the ascension of rising 66;19,36°, and the degree of rising 3^s 1;3°.

12r: 5–6 *Back of the Greater Bear* (*ẓahr dubb al-akbar*): The ascension of transit should be 155;2° instead of 115;2° (scribal error), but the degree of transit was calculated from the incorrect ascension (its correct value would be 4^s 23°).

13r: 1–4 *Shoulder of the Lion* (*mankib al-asad*): The ascension of transit should be 146;26° instead of 147;26° (scribal mistake), but the incorrect value in the text was used for the calculation of the rising point. The degree of transit for the text value of the ascension is 4^s 25;9°. Also the sine of the equation of daylight is incorrect; the tangent of the distance from the equator should be 24;54 instead of 24;15 (scribal error?), the lowered product of the two tangents 24;2,32, the equation of daylight 23;37,20°, the ascension of the rising point 122;49°, and the degree of the rising point 4^s 16;23°.

12v: 8–11 *Back of the Lion* (*ẓahr al-asad*): This star does in fact rise and set. Its equation of daylight is 24;17,46°, the ascension of the rising point 136;5°, and the degree of the rising point 4^s 26;26°.

12v: 19–21 *Braided Lock of the Lion* (*ṣafra* [for *ḍafīra*] *al-asad*): This star does rise and set. Its equation of daylight is 40;7,16°, the ascension of the rising point 138;19°, and the degree of the rising point 4^s 28;8°.

12v: 21–24 *Leader* (*al-qāʾid*): The non-existing sine 74;5,25 points to the use of radius of the base circle 60 for the trigonometric functions. It is the lowered product of the tangents 57;56 and 1,16;44.

13r: 1–4 *[Right] Wing of the Raven* (*janāḥ al-ghurāb*): The equation of daylight was calculated as the arcsine of 15;0,2 instead of 15;0,52 (reading mistake?).

13r: 5–8 *Unarmed Simāk* (*simāk al-aʿzal*): The ascension of the rising point should be 201;21°, and the degree of the rising point 6^s 16;22°.

13r: 13–16 *Brightest in the Beggar's Bowl* (*nayyir al-fakka*): The ascension of the rising point should be 194;46°, for which the degree of the rising point becomes 6^s 11;20°.

13r: 24–27 *Neck of the Snake* (*ḥunq al-ḥayya*): The omitted value for the equation of daylight should be 8;10,21°.

13r: 27–13v: 3 *Dancer* (*al-rāqiṣ*): The given tangent of the distance from the equator, 87;40, is exact; it again points to the use of radius of the base circle 60 for the trigonometric functions.

13v: 8–11 *Forehead of the Scorpion* (*jabhat al-ʿaqrab*): The equation of daylight should be 19;0,8°, but the following calculations were made on the basis of the value in the text.

13v: 19–22 *Head of the Kneeling One* (*raʾs al-jāthī*): The ascension of transit should be 250;55° instead of 255;55° (scribal error), but all following calculations were made on the basis of the value in the text.

Point of the Arrow (*zuḥḥ al-nushshā[ba]*): The ascension of transit would be slightly more accurate for the latitude −6;30 found in some Greek and Arabic sources, but the tangent of the distance from the equator would be much worse. The omitted value for the equation of daylight is 33;21,19°. 14r: 9–12

Eye of the Archer (*ʿayn al-rāmī*): The sine of the equation of daylight is given instead of the equation itself, which should be 23;48,25°. 14r: 13–15

Falling Vulture (*al-nasr al-wāqiʿ*): The equation of daylight was completely miscalculated; it should have been 50;22,51°. However, as in most cases, the value in the text was used for the remainder of the calculations. 14r: 16–19

Achilles Tendon of the Archer (*ʿurqūb al-rāmī*): The ascension of transit should be 278;47° as in the table on fol. 20r, but the degree of transit was calculated from the incorrect value in the text. 14r: 20–24

Flying Vulture (*al-nasr al-ṭāʾir*): The correct degree of the rising point is 8^s 19;31°. 14r: 25–27

Beak of the Fowl (*minqār al-dajāja*): The correct value of the equation of daylight calculated from the given sine is 29;44,54°; the following ascension of the rising point was in fact computed on the basis of an equation of daylight of 29;45°. 14v: 1–4

Tail of the Dolphin (*dhanab al-dulfīn*): The correct value of the tangent of the distance from the equator is 10;11, but the calculator apparently misread this as 10;51, which he used for the multiplication by the tangent of the latitude. 14v: 5–8

Raised Part of the Kid's Tail (*shawla dhanab al-jadī*): The correct value of the equation of daylight is 20;16° instead of 20;26°. 14v: 9–12

Mouth of the [Southern] Fish (*fam al-ḥūt*): The lowered product of the two tangents is incorrect (it should have been 39;33,20). The equation of daylight was apparently found as the arcsine of 40;13,57 (its correct value would have been 41;15,26°). 14v: 17–20

Rear [of the Fowl] (*al-ridf*): The missing degree of transit is 10^s 3°. The tangent of the distance from the equator was taken to be 56;0, whereas its accurate value is 57;1 (scribal confusion?). The latter value would have led to an equation of daylight of 66;34,12°, an ascension of the rising point of 239;16°, and a degree of the rising point of 7^s 15;12°. 14v: 21–24

Northern Star in the Whale's Tail (*dhanab qīṭs shamālī*): The product of the tangent of the distance from the equator, given as 12;16, and the tangent of the latitude, 57;56, is here presented to four sexagesimal places that are all correct (in most cases the product is rounded to three digits). An accurate value for the degree of the rising point is 0^s 16;54°. 15r: 9–12

Southern Star in the Whale's Tail (*dhanab qīṭs janūbī*): The tangent of the distance from the equator can here be reconstructed from the given equation of daylight, 21;26,32°. The sine of the equation is 21;56,2, which is very close to the lowered product of the value for the tangent of the latitude used for all fixed star calculations, 57;56, and 22;43 (correct value: 22;42). 15r: 13–15

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Appendix A: Chinese-Uighur Calendar

2r: 25–2v: 3 This Appendix to Section 3 supplements the conclusions concerning the Chinese-Uighur date in the Horoscope of Iskandar reached in Elwell-Sutton, 'A Royal Tīmūrid Nativity Book', pp. 121–123, with a mathematical analysis based on the discussion in van Dalen et al., 'The Chinese-Uighur Calendar'.

In Chinese calendars a day consists of twelve double-hours (Turkish *chāgh*, Chinese *shi* 時), which were further divided into eight quarters of an hour (Persian transcription *kih*, Chinese *ke* 刻). The double-hours were counted by the elements of the duodecimal animal cycle and the first double-hour started at 11 pm. A day was also divided into ten thousand 'parts' (Persian transcription *fing*, Chinese *fen* 分), so that each quarter of an hour equaled $104\frac{1}{6}$ *fen*s. Since the indications *khāy* (Chinese *hai* 亥) and *ṭunghūz* (Turkish *tonguz*, 'pig') denote the last element of the duodecimal cycle, the Chinese-Uighur time given in the Horoscope corresponds to $\frac{1}{15}$ *fen* before the end of the day, i.e., slightly more than half a second before 11 pm. It is unclear why the time of the nativity is not simply given as the beginning of the following Chinese day, especially since 'four equal hours after sunrise' (cf. above) was in fact a little later than 11 pm. Possibly the following Chinese-Uighur day was a less auspicious one.

The day of the nativity is said to be the eighth of the sexagesimal cycle of the Chinese-Uighur calendar, whose elements were enumerated by a combination of those of the animal cycle and the decimal cycle of heavenly stems. The indication *sin-way* (Chinese *xin-wei* 辛未) combines the eighth elements of both cycles and hence indicates the eighth element of the sexagesimal cycle; *qūy* (Turkish *qoy*, 'sheep') denotes the eighth element of the animal cycle. The sexagesimal day-count, which is similar to our week, has been consistently used in China over a period of thousands of years, and Sunday, 24 April 1384, was in fact a day 8 of the cycle. In an independent 'fourth cycle', named the 'cycle of choices' (*dawr-i ikhtiyārāt*), this day is said to correspond to the fourth of twelve elements, *pin* (Chinese *ping* 平), which has the attribute *khay* (Chinese *hei* 黑, 'black') associated with corruption. Since the next element of the cycle of choices is 'inclined toward beneficence', my suggestion above that the eighth day of the sexagesimal cycle is given as the date of birth since it might be less auspicious than the ninth day seems to be invalidated.

There follow two numbers with fractions indicating the exact position of the time of birth in the two main defining astronomical cycles of the Chinese-Uighur calendar, namely true lunar months and the mean solar year. The true new Moon indicating the beginning of the current Chinese month, namely, the fourth month *dūrdinch āy* (Turkish *Törtünc ay*), preceded the time of the nativity by exactly three days and 4215 (i.e., 1,10,15) *fen*s. Furthermore, the beginning of the current solar half-month (Persian *qism*, Chinese *qi* 氣, a twenty-fourth of the mean solar year), namely the sixth half-month *kūwū* (Chinese *guyu* 穀雨, 'grain rain'), preceded the time of the nativity by ten days and 8677 (i.e., 2,24,37) *fen*s. A recomputation of these two numbers from the rules given in the *Īlkhānī Zīj* yields 3 days 4634 *fen*s and 10 days 9096 *fen*s respectively, both 419 *fen*s larger than the values in the text. The difference might be due to an adjustment for a longitude difference of 15;5° in eastern direction (since the local time of the true new Moon and the beginning of the *qi* is later in a locality further east). However, at first sight there are no plausible localities 15;5° east of Maragha or 15;5° west of Uzgand for or from which the adjustment could have been made. It must also be noted that no correction for geographical longitudes different from Maragha is prescribed in the *Īlkhānī Zīj*, and that Chinese calendars in general were devised for one particular central locality. The reason for the subtraction of 419 *fen*s therefore remains unclear.

Also the years in Chinese calendars are counted by the sexagesimal cycle. In the Uighur variant three consecutive cycles of 60 years are called *shāng win*, *jūng win* and *khā win* (Chinese *shangyuan* 上元, *zhongyuan* 中元 and *xiayuan* 下元). Since the *Īlkhānī Zīj* makes 1264 CE the first year of a *shangyuan*, Iskandar's year of birth (1384) is in fact the first year of a *xiayuan* and is hence correctly indicated by *kā zhih* (Chinese *jia-zi* 甲子, first element of the sexagesimal cycle), *kuskū* (Turkish *küskü*, 'rat'), or *sīchqān* (Turkish *sıçan*, 'rat'). The years since the creation of the world are counted in *wan*s (Chinese *wan* 萬, 'ten thousand') of ten thousand years. The indication that, in the year of the nativity, 8863 *wan*s and 9860 years of the current *wan* had been completed is in full agreement with the *Īlkhānī Zīj* (and hence with the Chinese *Damingli*, which had been adopted by the Mongols when they first conquered the Jin dynasty in 1215 CE).

Appendix B: The Characteristics of the Underlying Planetary Tables

This Appendix to Section 7 discusses in detail how the characteristics of the planetary tables used for the calculation of the Horoscope of Iskandar Sultan, in particular in relation to the use of displacements, can be determined from the planetary data contained in it. It furthermore shows that the tables in the *Īlkhānī Zīj* have precisely these characteristics.

We may note the following peculiarities in the intermediate steps of the computations of planetary longitudes in the Horoscope:

a) Many of the equations at the times of the nativity and of conception are clearly larger than their regular maximum values:

1. solar equation (nativity): 3;29,58° (maximum normally around 2°) 2v: 12
2. lunar equation of centre (nativity): 24;52,30° (maximum usually 13;8°) 4r: 10
3. Venus equation of centre (nativity): 3;12,23° (maximum around 2°) 2v: 25
4. Venus equation of anomaly (nativity): 10^{s} 14;56,21° (maximum near 45°) 2v: 27
5. Jupiter equation of centre (conception): 9;21° (maximum 5;15°)[81] 16r: 8

[81] This equation was omitted from the manuscript, but it follows directly as the difference of the given mean anomaly 2^{s} 26;27° and the true anomaly 2^{s} 17;6°.

16r: 11 6. Mars equation of centre (conception): 23;21° (maximum around 11;25°)
16r: 19 7. Mercury equation of centre (conception): 7;2° (maximum around 3;2°)
16r: 4 8. lunar equation of anomaly (conception): 12;35,1° (maximum usually 7;40°)
16r: 7 9. Saturn equation of anomaly (conception): 11;44° (maximum around 6;36°)
16r: 10 10. Jupiter equation of anomaly (conception): 22;31° (maximum near 11;36°)
16r: 22 11. Mercury equation of anomaly (conception): 11^{s} 21;42° (maximum 23;53°)

b) In Ptolemaic planetary tables set up in the traditional way, as, for example, in the *Almagest*, *Handy Tables*, and the early Islamic zījes of Yaḥyā ibn Abī Manṣūr, al-Battānī and Ibn Yūnus, the equations need to be added to, or subtracted from, certain quantities depending on the values of their arguments. However, in the Horoscope of Iskandar the planetary equation of centre is *always* added to the mean centrum and *always* subtracted from the mean anomaly; all other equations are *always* added to the appropriate mean motions.

5r: 11, 14 **c)** The instructions for calculating the planetary latitudes prescribe an addition of 7° to the 'adjusted centrum' (*markaz-i muʿaddal*) for Saturn in order to obtain the 'actual adjusted centrum' (*markaz-i muʿaddal-i ḥaqīqī*), and a similar addition of 12° to the adjusted centrum for Jupiter.

Displaced Planetary Equations

All three characteristics mentioned above clearly point to the use of so-called 'displaced equations'. By increasing the equations by a certain constant positive amount, many Islamic astronomers made them additive (or, in some cases, subtractive) for *all* arguments rather than additive for a certain range of arguments and subtractive for another. Usually the amount added to the equations was equal to the maximum equation or the next higher integer degree of that maximum. For example, a displaced version of the equation of centre q for Saturn, which usually has a maximum value of 6;31°, might be defined by $7° + q$ for additive values of the equation and $7° - q$ for subtractive values.[82]

The introduction of displacements has various consequences for the tables of mean motions and equations. As an example, let us consider a displacement of the equation of centre q_c for a certain planet by an amount $d_c > 0$, and of the equation of anomaly q_a for the same planet by an amount $d_a > 0$. Normally q_c is subtracted from, or added to, the mean anomaly in order to obtain the true anomaly. However, in the case of a displaced equation of centre a quantity $d_c + q_c$, respectively a quantity $d_c - q_c$ will be subtracted. Therefore, the mean anomaly as found in mean motion tables to be used with this displaced equation of centre should be *larger* than the actual mean anomaly by the amount d_c in order to produce the correct true anomaly. Similarly, since both the displaced equation of centre and the displaced equation of anomaly will be added to the

[82] References to more extensive discussions of displaced equations are given in footnote 27 on p. 23.

mean centrum,[83] the mean centrum as found in the mean motion tables should be *smaller* than the actual mean centrum by the amount $d_c + d_a$ in order to yield the correct true centrum when the two equations are added.

However, the mean centrum is also the argument from which the equation of centre is found. Most conveniently, we want to be able to enter the table for the equation of centre with the same value for the mean centrum that we have found from the mean motion tables. Since this value is reduced by the amount $d_c + d_a$, this implies that the tabular values for the equation of centre must be 'shifted' backwards by this same amount, i.e., in the table the equation of centre for the actual mean centrum $\bar{c}$ should appear next to the displaced mean centrum $\bar{c} - (d_c + d_a)$. Since most equations are tabulated for every degree of the argument, it is thus more practical to make the displacements equal to integer numbers. Therefore they are mostly taken equal to the nearest integer larger than the maximum equations.

The Displacements in the Horoscope of Iskandar

Let us now try to determine the amounts by which the tables used for the calculation of the planetary longitudes in the Horoscope of Iskandar were displaced. Firstly, as mentioned under c) above, respectively 7° and 12° must be added to the found 'adjusted centrum' of Saturn and Jupiter in order to obtain the 'actual adjusted centrum', which is needed for calculating the latitudes of these planets. From the above discussion it now follows that these numbers are very probably the displacements d_a of the equations of anomaly (since the equation of centre is already incorporated in the adjusted centrum, it is not necessary to also add d_c in order to obtain the actual adjusted centrum). Note that 7° and 12° are plausible displacements for Saturn and Jupiter, because the maximum equations of anomaly for these planets are approximately 6;36° and 11;36°. Since apparently similar adjustments were not necessary for the true centrums of Mars, Venus and Mercury, it seems that the equation of anomaly of those planets was not displaced. However, of the five values for the equation of anomaly for these three planets that are found in the Horoscope, the ones for Venus and Mercury listed above are much larger than the ordinary maximum equations concerned. It can be verified that both involve the special type of displacement that I have called 'displacement by twelve zodiacal signs'.[84] This means that a subtractive equation of anomaly q is represented as $12^s - q$, so that it can always be added to the 'adjusted centrum' in order to obtain the

[83] In some zījes the planetary equation of anomaly is added to the mean longitude rather than to the mean centrum (the difference between the two being the longitude of the planetary apogee) in order to obtain the true longitude directly. In such cases the tabulated mean longitude should be smaller than the actual mean longitude by an amount d_a and the tabular values of the equation of centre only need to be shifted backwards by an amount d_c, instead of by $d_c + d_a$ (cf. below).

[84] See van Dalen, 'The *Zīj-i Nāṣirī*', p. 841.

'corrected centrum' without necessitating a (further) adjustment of the mean centrum (however, in many cases, 12 signs will need to be subtracted in order to obtain a result between 0 and 360°). We may assume that also the equation of anomaly for Mars was of this type.

In trying to determine the displacements of the underlying planetary equations of centre from the values given in the Horoscope, we need to take into account that the values for the mean centrum from which they were found were also displaced. As usual, natural candidates for the amounts of the displacement are the next larger integers to the maximum equations. Since the equations change relatively slowly in the neighbourhood of their maxima, it may be useful to first look at planets for which the given equations of centre are close to their maximum or, equivalently, for which the mean centrum is relatively close to 90° or 270°. For example, the mean centrum of Mercury at
16r: 19 the time of conception is given as $8^{s}\,21;9^{\circ}$. For Ptolemy's value for the eccentricity of Mercury, 3;0, the corresponding equation is +3;1°, which differs by almost precisely 4;0° from the text's 7;2°. Anticipating a displacement of 4° (i.e., the next larger integer to Mercury's maximum equation of centre of 3;2°), the given mean centrum corresponds to an actual mean centrum of $8^{s}\,25;9^{\circ}$, for which Ptolemy's equation becomes +3;2°, differing by precisely 4° from the value in the text. We conclude that the table of the equation of centre for Mercury used in the Horoscope was displaced by 4°.

A similar reasoning works quite well for the equation of centre for Mars
16r: 11 at the time of conception. Here the displaced mean centrum is $8^{s}\,19;59^{\circ}$, the equation of centre in the text 23;21°, and Ptolemy's equation +11;23°. The difference, 11;58°, is very close to the next larger integer of Ptolemy's maximum equation, 11;25°. In this case the agreement of the text value with Ptolemy's value for the assumed actual mean centrum of $9^{s}\,1;59^{\circ}$, namely +11;19°, is not as good as for Mercury. However, errors of 2′ in medieval equation tables are not uncommon. Thus we conclude that the displacement of Mars's equation of centre is 12°.

For Saturn, Jupiter and Venus some trial-and-error is needed, since none of the given equations in the text are close to their maximum. The given mean centrums for Saturn and Jupiter can already be corrected for the displacement of the equation of anomaly found above before sample equations are calculated. For example, for Saturn at the time of conception we find: mean centrum in
16r: 6 the text $5^{s}\,13;41^{\circ}$, equation in the text 6;43°, mean centrum adjusted for the displacement of the equation of anomaly $5^{s}\,20;41^{\circ}$, Ptolemaic equation −1;7° (maximum 6;31°), difference 7;50°. Here the displacement turns out to be 7° (rather than 8°), since for 7° the equation for the actual mean centrum $5^{s}\,27;41^{\circ}$ becomes −0;17°, differing from the text by precisely 7°. In a similar way we find a displacement of 6° for Jupiter.

For the equation of centre for Venus we cannot simply assume the use of Ptolemy's value for the eccentricity, because it was drastically improved upon

by Islamic astronomers. However, since in most zījes the maximum equation for Venus was close to 2°, we can still determine the displacement with reasonable certainty. For the given mean centrum at the time of the nativity, 10^{s} 19;18,25°, the text gives the equation of centre as 3;12,23°. Assuming that 2v: 25
the maximum equation is precisely 2°, a recomputation yields +1;17,13°, differing by 1;55,10° from the text. Anticipating a displacement of 2°, we note that the recomputed equation for 10^{s} 21;18,25° is 1;14,0°, differing by 1;58,23° from the equation in the text. However, a similar procedure for the equation of centre for Venus at the time of conception, namely 1;47°, rather points to a displacement of 3°, since the recomputed equation for the mean centrum in the text is −1;10,29°. It thus seems that one of the two calculations in the text involves some rather large errors.

The situation for the Moon is slightly simpler than for the planets. Since the equation of centre is only added to the mean anomaly, and the equation of anomaly to the mean longitude rather than to the mean centrum, the equation of centre does not need to be shifted. This implies, however, that displacements by non-integer numbers of degrees, in particular by the maximum equations, are possible. For the Moon, most Islamic astronomers likewise used Ptolemy's eccentricity and radius of the epicycle or slight variations of them. For the time of the nativity, the lunar mean centrum is given in the Horoscope as 2^{s} 27;5,16° and the equation of centre as 24;57,30°, whereas Ptolemy's equation 4r: 10
is 11;43,24°, differing by 13;14,6° from the value in the text. For the time of conception, the mean centrum in the Horoscope is 9^{s} 22;33,49° and the equation 4r: 22, 16r: 3
of centre 3;35,37°; the calculated equation is −9;32,29°, differing by 13;8,6° from the text. It is thus plausible that the underlying lunar equation of centre was displaced by its maximum, 13;8°. This also implies that the tabulated mean anomaly is 13;8° smaller than the actual one.

The lunar mean anomaly at the time of the nativity is given in the Horoscope as 1^{s} 13;58,22° and the second equation as 2;45,30°. Since the second 4r: 10–11
equation could be either the equation of anomaly at apogee or that at perigee (the two functions may even be combined in a single table), we need to compare the text value with both possibilities. Ptolemy's recomputed equation of anomaly at apogee becomes −3;16,16° (difference from the text value 6;1,46°), the equation at perigee −4;49,45° (difference 7;35,15°). The true anomaly at the time of conception is given in the Horoscope as 7^{s} 24;37,46° and the second 4r: 23, 16r: 4
equation as 11;57,52°. Here Ptolemy's recomputed equation at apogee becomes +4;17,53° (difference from the text 7;39,59°), that at perigee +6;43,15° (difference 5;14,37°). As possible displacements we expect the maximum equations (approximately 5° at apogee, 7;40° at perigee) or next higher integers (5°, 6° or 8°). Judging from the above differences between text and recomputations, the most plausible displacement is thus 7;40°, in which case the second equation given in the text for the time of the nativity (with an argument between 0° and 180°) is an equation at perigee, whereas the second equation for the time of

conception (with an argument between 180° and 360°) is an equation at apogee (for an explanation of this 'phenomenon', see below).

For the Sun, a range of values for the eccentricity is historically plausible, and the maximum solar equation may be slightly smaller or slightly larger than
2v: 12 2°. For the time of the nativity the mean centrum is given as $10^{s}\,10;34,38°$ and the equation as 3;29,58°. For the time of conception the mean centrum is
16r: 2 $0^{s}\,27;48,44°$ and the solar equation 1;5,44°. Calculating with a maximum solar equation of precisely 2° we find non-displaced equations equal to +1;29,5° and −0;54,18° respectively, differing by 2;0,53° and 2;0,2° from the text. It is thus clear that the displacement will be either precisely 2° or equal to the maximum equation. Since the unknown maximum equation $q_{\max}$ can be assumed to lie in a rather narrow interval around 2°, we can approximate the calculated non-displaced equations quite well by $+0;44,32 \cdot q_{\max}$ for the time of the nativity and $-0;27,9 \cdot q_{\max}$ for the time of conception. If the displacement is exactly 2°, we expect these to be equal to 3;29,58° − 2° = 1;29,58° and 1;5,44° − 2° = −0;54,16° respectively. This leads to inconsistent maximum equations of 2;1,13° and 1;59,56°. If the displacement is equal to the maximum equation, we expect the approximating expressions for the equations to be equal to $3;29,58° - q_{\max}$ at the time of the nativity and $1;5,44° - q_{\max}$ at the time of conception. Thus we find maximum equations of approximately 2;0,31° and 2;0,4°, somewhat less inconsistent than the above pair of values. Although we can not yet decide with certainty, it is clear that the maximum solar equation lies close to 2° and that the displacement is either 2° or equal to the maximum equation.

We have thus found the following displacements in the planetary tables that were used for the calculation of the longitudes in the Horoscope of Iskandar: *solar equation*: close to 2°; *equations of centre*: Moon 13;8°, Saturn 7°, Jupiter 6°, Mars 12°, Venus 2° or 3°, Mercury 4°; *equations of anomaly*: Moon 7;40°, Saturn 7°, Jupiter 12°, Mars 12^{s}, Venus 12^{s}, Mercury 12^{s}. No complete overview of the displacements of the planetary tables in Islamic zījes is available yet, but of around twenty important zījes that I have checked there is only one that has the above-mentioned displacements, namely the *Īlkhānī Zīj* by Naṣīr al-Dīn al-Ṭūsī. Although the same displacements of the planetary equations of centre are already found in Kūshyār b. Labbān's *Jāmiʿ Zīj* (*c.* 1025),[85] in particular the combination of displacements 7° and 12° for the equation of anomaly of Saturn and Jupiter and 12^{s} for the other planets is very rare.

As far as the displacements are concerned that we could not establish with certainty from the data in the Horoscope, in the *Īlkhānī Zīj* the solar equation is displaced by its maximum, 2;0,30°,[86] and the equation of centre of Venus

[85] See Van Brummelen, 'Mathematical Methods', pp. 268–271, and van Dalen, *Ptolemaic Tradition*, pp. 408–410.

[86] The displacement 2;0,30 of the solar equation occurs in most of the manuscripts of the *Īlkhānī Zīj* that I have consulted, in particular in Cairo, Dār al-kutub, *mīqāt fārisī* 1

by 2°. Al-Ṭūsī's table for the lunar equation of anomaly is of so-called 'mixed type',[87] which means that the equation at apogee is tabulated in one half of the table and the equation at perigee in the other half (a property that we had already noticed above for the two values of the lunar equation of anomaly presented in the Horoscope). This also explains why the product of 'difference' and interpolation minutes can be added to the second equation in both instances in the Horoscope, whereas for ordinary displaced equation of anomaly tables it would have to be added in one half of the epicycle and subtracted in the other.

Appendix C: Equation of time

True time is determined directly from solar observations and is defined in such a way that every day the Sun culminates precisely at noon. Due to the inclination of the ecliptic to the equator (in modern, heliocentric terms: the tilt of the Earth axis with respect to the plane of the Earth's motion around the Sun) and the eccentricity of the solar orbit around the Earth (in modern terms: the eccentricity of the Earth's path around the Sun), the time between true noons defined in this way is not constant. On the other hand, *mean time* is determined in such a way that the time between two consecutive mean noons is always the same (namely, 24 hours). Mean time is easier to use in calculations and is utilised in planetary tables to facilitate the calculation of planetary mean motions. The difference between true and mean time is called the *equation of time* (Arabic: *taʿdīl al-ayyām wa-layālīhā*) and was already tabulated by Ptolemy in his *Handy Tables*. It needs to be taken into account in particular when calculating accurate lunar positions and the times and magnitudes of eclipses.[88]

The *Īlkhānī Zīj* contains tables for the corrections of solar and lunar longitudes due to the equation of time in its treatise on planetary positions and a table expressing the equation in minutes and seconds of an hour (as found in most other zījes) in the treatise on spherical astronomy.[89] Each of these tables has degrees of the true solar longitude as its argument, and the given values of the

(fols 34v–37r); Istanbul, University Library, F 1418 (fols 130v–133r); Leiden, Universiteitsbibliotheek, Or. 75 (fols 33r–35v); and Florence, Biblioteca Medicea Laurenziana, Or. 24 (fols 30r–32v). In these manuscripts the longitude of the apogee, rather than the mean solar centrum, was decreased by the displacement in order to yield the correct true longitude after the addition of the solar equation and the longitude of the apogee. In at least one other manuscript of the *Īlkhānī Zīj*, namely, the very early Paris, Bibliothèque nationale de France, persan 163, which was copied by al-Ṭūsī's son Aṣīl al-Dīn Ḥasan, a different solar equation table is found with a displacement *and* a shift of precisely 3° (fols 27v–29r). However, the resulting solar longitudes are the same in each case.

[87] cf. van Dalen, 'The *Zīj-i Nāṣirī*', pp. 842–843.

[88] For extensive discussions of the equation of time and further references, see van Dalen, 'On Ptolemy's Table', and van Dalen, 'Al-Khwārizmī's Astronomical Tables'.

[89] The tables can be found, for example, in Paris, Bibliothèque nationale de France, persan 163, fols 29v (Sun), 36v (Moon), and 123r (time, misplaced after Treatise IV).

equation of time must be subtracted from (the positions at) true solar time in order to obtain (the positions at) mean solar time. Since the true solar longitude as computed from the solar tables is generally not a whole number of degrees, linear interpolation needs to be applied to find the equation of time from one of the three tables. The tables are based on al-Ṭūsī's known values 23;30° for the obliquity of the ecliptic and 2;6,9 for the solar eccentricity, and on a longitude of the solar apogee equal to 90°.

2v: 13 ʿImād al-munajjim Maḥmūd al-Kāshī first applies the equation of time when he determines a preliminary value of the ascendant at the time of Iskandar's birth. For this he subtracts an equation of 0;0,50° (in agreement with al-Ṭūsī's solar table) from the true solar longitude at the observed time of birth (1^{s} 12;39,35°) to obtain the longitude at the mean time of birth.

4r: 19, 16r: 2 Next he finds the solar equation of time at mean midnight of the day of conception as 0;0,28° (again in agreement with the table), but he mistakenly subtracts it from the true solar longitude calculated for mean midnight (3^{s} 27;25,13°), rather than adding it in order to obtain the true solar longitude at true midnight (which is equal to upper midheaven, from which he can then easily calculate the ascendant). The error is of the order of 4 seconds of time, so that it does not have any practical consequences.

4r: 21–22 Finally, al-Kāshī applies the equation of time implicitly when he converts the true time of conception calculated from the preliminary ascendant at birth (1;46,23,52 hours after midnight) to mean time (13;35,10,52 hours after noon; see p. 51). The difference of $0^{h}11^{m}13^{s}$ is again in exact agreement with al-Ṭūsī's table (it is the result of linear interpolation between the values 0;11,15 for $3^{s}\,27°$and 0;11,11 for $3^{s}\,28°$).

4r: 12 In two calculations of lunar positions al-Kāshī gives the equation of time
4r: 25 as 0;0,0° although the corresponding tabular values are non-zero. However, in both cases the positions are calculated and needed for mean time, so that no correction is in fact necessary.

Appendix D: Correction of the Ascendant (*namūdār*s)

According to the *namūdār* of Hermes (cf. 2] in Section 14) the lunar longitude at the time of the nativity is equal to the longitude of the ascendant at the time of conception, and the other way around. In order to find a corrected value for the ascendant (and hence for the time of the nativity) from an initial estimate of the time of birth, first an approximation to the time of conception is determined by means of a formula for the duration of gestation. The mean period of gestation is taken to be equal to exactly ten lunar cycles in longitude, i.e., $273^{d}5^{h}12^{m}$.[90] This mean period is corrected by the 'equation of gestation', which is defined as the difference in longitude between the ascendant and the

[90] Ghiyāth al-Dīn al-Kāshī and other Islamic authors also consider the possibility of shorter and longer pregnancies; see Kennedy, 'Treatise V', p. 142.

true lunar position at the estimated time of birth divided by the mean daily lunar motion in longitude. If the Moon is below the horizon at the time of birth, the equation must be added to the mean period in order to find the preliminary time of conception; if it is above the horizon, it must be substracted. Thus the preliminary time of Iskandar's conception can be found as approximately $19^{h}30^{m}$ on 13 July 1383 (Maḥmūd al-Kāshī does not mention an explicit time but would have arrived at 21^{h} with the period of gestation given in the text).

This procedure provides a very reasonable approximation to the time of conception according to the *namūdār* of Hermes, because the equation gives the period of time (either before or after the nativity) that the Moon needs to reach the longitude of the ascendant at the time of birth under the assumption that its longitude changes at a linear rate. The amount by which the lunar longitude at the obtained preliminary time of conception differs from the ascendant at the time of birth is only due to the difference in equation of anomaly between the estimated time of the nativity and the obtained preliminary time of conception Since the maximum equation of anomaly amounts to 7;40°, the maximum possible difference in equation of anomaly between two arbitrary points in time is 15;20°, somewhat larger than the daily lunar motion in longitude, so that it would require an adjustment of the preliminary time of conception of little more than a day. At the estimated time of birth of Iskandar the lunar equation of anomaly was +3;57,22° (with displacement: 3;42,38°), while at the time of 4r: 11
conception it was −4;55,1°(with displacement: 12;35,1°). The difference be- 4r: 24
tween the two equations, 8;52,23°, corresponds to approximately $16^{h}10^{m}$ in lunar mean motion, which implies that the lunar longitude was close to the ascendant at the estimated time of birth at around $3^{h}20^{m}$ on 13 July. In fact, at this time the lunar position according to the *Īlkhānī Zīj* was 270;35°, differing by only 0;21° from the estimated ascendant 0;56° Capricorn.[91]

Next the author of the Horoscope of Iskandar finds the time of conception by determining the moment when the ascendant is exactly equal to the lunar longitude at the preliminary time of birth (see 3] in Section 13). Finally, he makes the corrected ascendant at the time of birth equal to the lunar longitude at the time of conception. This procedure can be seen as follows to have an exact solution. Let a value $A_b(t_b)$ for the ascendant at an arbitrary point in time t_b be given, and let $M_b(t_b)$ be the true lunar position at this time. Let t_c be a different point in time at which the true lunar position $M_c(t_c)$ is always equal to $A_b(t_b)$. Now let t_b increase at an arbitrary rate. Since the ascendant changes at a rate which, on the average, is 360/13;10,35 $\approx$ 27.3 times as large as that of the moon, t_c needs to increase at a rate approximately 27.3 times as large as t_b for $M_c(t_c)$ to stay equal to $A_b(t_b)$. At the same time, the corresponding

[91] Note that it is a coincidence that in Iskandar's case the exact time of conception ($1^{h}35^{m}11^{s}$; see p. 51) differs from the time when the lunar longitude is equal to the estimated ascendant by less than two hours: since each degree of the ecliptic rises only once during every 24 hours, it could just as well have differed by twelve hours.

ascendant $A_c(t_c)$ increases at a rate approximately 27.3 times as large as $M_c(t_c)$. On the other hand, the lunar longitude $M_b(t_b)$ increases at a rate approximately 27.3 times as slow as $A_b(t_b)$. Overall, $A_c(t_c)$ increases at a rate approximately $27.3^2 \approx 746.5$ times faster than $M_b(t_b)$ and will inevitably overtake it, thus providing an exact solution to the *namūdār* of Hermes.

Since the ascendant and the lunar position at the corrected time of birth will be somewhat different from those at the originally estimated time, the criterion of Hermes will not hold exactly at the newly found time. It can be shown as follows that the procedure used in the Horoscope, if applied iteratively, converges.[92] Let A_b^0 and M_b^0 be the initial values of the ascendant and the lunar longitude at the estimated time of birth, and choose an appropriate day during which the lunar longitude assumes the value A_b^0. On this day, determine the time at which the ascendant A_c^0 is equal to M_b^0 and calculate the lunar longitude M_c^0 at that time. Since A_c^0 may differ from A_b^0 by up to $180°$, M_c^0 may differ from A_b^0 by half the daily lunar motion, i.e., by at most around 7.5 degrees. As a result, the corrected estimate A_b^1 of the ascendant at the time of birth, which is taken equal to M_c^0, also differs from A_b^0 by at most around 7.5 degrees. The corresponding lunar longitude M_b^1 may then differ from M_b^0 by approximately a 27th of this amount (cf. the previous paragraph), i.e., by at most 0.3 degrees. Consequently, in the next step, A_c^1 will differ from A_c^0 by at most around 0.3 degrees, so that M_c^1 will differ from M_c^0 (and hence A_b^2 from A_b^1) by at most around $0.3/27 \approx 0.011$ degrees. We conclude that this iteration converges to a solution of the *namūdār* of Hermes and that, in practice, the first step is sufficient to obtain the ascendant satisfying the criterion to an accuracy of minutes.

Not every value of the ascendant can be a solution of the *namūdār* of Hermes. Depending on the exact way in which the day of conception is defined,[93] for any given day of birth there are around 27 possible values of the ascendant that satisfy the criterion. These values are unevenly distributed over the day with intervals that are inversely proportional to the rate of change of the ascendant. They are separated by the day of conception, i.e., for all values of the lunar longitude reached on the same day of conception, the ascendant according to the *namūdār* of Hermes will be the same.

Appendix E: Prorogations (*tasyīr*s)

Prorogations are one of the most important means of predicting events in a native's life on the basis of the positions of the Sun, the Moon, the planets,

[92] Ghiyāth al-Dīn al-Kāshī, the grandson of the author of the Horoscope, in fact carries out the procedure iteratively; see Kennedy, 'Treatise V', pp. 140–143.

[93] Ghiyāth al-Dīn al-Kāshi states that, at the calculated time of conception, the lunar longitude should differ from the ascendant at the time of birth by less than half a day's travel of the moon; see Kennedy, 'Treatise V', p. 142. Another possibility is to make the day of conception the 24-hour period (reckoned from noon or midnight) during which the lunar longitude reaches the position of the ascendant at birth.

the cusps of the houses, and the astrological lots at the time of birth. They were called ἄφεσις in Greek, *aphesis*, *atazir* or *directio* in Latin, and are also referred to as 'progressions' or '(primary) directions' in English. A prorogation starts with a chosen point on the heavenly sphere (the prorogator), which is called *mutaqaddim(a)* in Arabic and Persian and *significator* in Latin. This point is assumed to move in the opposite direction of the daily motion of the sphere at a speed of one degree per year.[94] In the course of this motion the prorogator will reach significant configurations with other heavenly bodies or astrologically significant points, in particular conjunctions and aspects, on the basis of which predictions of favourable and adverse events can be made.

The *haylāj* (Latin: *hyleg*) is a special type of indicator that can be used as a prorogator to make predictions about health and the length of life. It is taken as the Sun, the Moon, the syzygy preceding the nativity, the lot of fortune, or the ascendant following a complicated selection process. The death of the individual is indicated by the prorogator coming under the influence of a so-called 'cutter' (*qāṭiʿ*; Latin: *promissor*). Primary candidates for the cutter are the malefic planets Saturn and Mars, but under certain circumstances also the Sun and the Moon.

Once the starting point of a prorogation was fixed, a variety of mathematical methods were used to calculate the progression of the *tasyīr*, and hence to determine by which indicators it would be influenced at which times.[95] ʿImād al-munajjim Maḥmūd al-Kāshī does not explain the calculation of prorogations in the text of the Horoscope, nor does he make an explicit use of numerical *tasyīr* arcs when he presents his predictions.[96] However, on fols 23v–62v he provides a large set of tables for prorogations for twenty possible starting points, namely the twelve houses, the seven planets and the lot of fortune. These tables have columns for: the solar year since Iskandar's birth (from 1 up to 88, written out in words); the prorogation (indicated in signs, degrees and minutes); the ruler

[94] Both in primary sources and in the secondary literature also the opposite interpretation is found: the prorogator is assumed to be fixed, and the celestial sphere with the heavenly bodies in their positions at the time of birth is then rotated in the direction of its regular daily motion in order to 'reach' the prorogator.

[95] The mathematical methods are classified in Casulleras & Hogendijk, 'Progressions, Rays and Houses'. For a general study of prorogations, see Gansten, *Primary Directions*. For the treatment by early Arabic authors, see Burnett et al., *Al-Qabīṣī (Alcabitius)*, pp. 120–129, and Yano, *Kūšyār Ibn Labbān's Introduction*, chs III.20–21, pp. 216–235. For further useful discussions of prorogations, see the article 'Tasyīr' by Oskar Schirmer in *The Encyclopaedia of Islam. New edition*, vol. X (2000), pp. 366–368 (with two pages with plates); Yano & Viladrich, 'Tasyīr Computation'; Díaz-Fajardo, *Tasyīr y proyección de rayos*; Gansten, 'Balbillus', and Hogendijk, 'Al-Bīrūnī on the Computation'.

[96] The section 'On the prorogations, termini, periods and ascendants of [year] transfers' on fol. 23r (just preceding the tables discussed here) only mentions the importance and the general use of prorogations for predictions. It mentions in lines 2–3 that the prorogations are based on the incidental horizon.

of the term (*al-qāsim*); basic indications (*al-dalāʾil al-aṣliyya*) / 'that which the prorogation passes' (*ānchih tasyīr bar ān gudharad*); annual termini (*intihāʾ*s, another type of indicator, not given for the ascendant), and the corresponding Malikī years (306 up to 393, only up to the sixth house) with the day of the year (always 15 Urdibihisht, only for the ascendant). The table for the lot of fortune has additional columns for *fardāriyya* and for the ascension of the ascendant of [year] transfers. The headers of the tables explicitly confirm that the prorogations were calculated according to the incidental horizons and mention the areas of the native's life about which the prorogators allow predictions (e.g., 'health and sickness' for the ascendant, and 'the sultanate, pride and shame' for the Sun). The column with indications lists situations that may have a positive or negative influence on the areas of the native's life covered by the prorogator. Examples include one of the other houses or planets, planetary conjunctions (indicated in red), fixed stars and their degrees of transit, and astrological lots.

For each of the twenty possible prorogators the annual terminus at the time of birth is equal to the longitude of the prorogator, and then increases by exactly one zodiacal sign per true solar year. The additional table on fols 63r–65r gives the termini of the ascendant for the months of the Malikī years in a twelve-year cycle, with an increase of approximately 2;27,51° per month.

As we have seen, the main purpose of the chapters of the Horoscope described in Sections 16 to 20 was to determine the point of intersection of the equator with the incidental horizon for a given planet (point *O* in Figure 18 on p. 67), which is determined by its 'corrected ascension', i.e., its distance from the vernal equinox. The longitude of the intersection of the ecliptic and the incidental horizon is called the 'corrected degree' and can be found from the corrected ascension by means of an inverse look-up in the oblique ascension table for the incidental latitude. In the tables of prorogations in the Horoscope, the initial prorogation at the time of birth is generally equal to the corrected degree of the heavenly body concerned. These degrees were omitted from the Horoscope, but the results of my own computations on p. 72 generally agree quite well with the initial prorogations in the tables. This is particularly significant for the Moon, since its large latitude of nearly five degrees leads to a similarly large difference between its ecliptic longitude and the corrected degree.

We may now expect the further prorogations to have been found as the inverse oblique ascension for the incidental latitude of the corrected ascensions increased by one equatorial degree for every true solar year. However, in a number of cases that I have checked, this assumption did not lead to an acceptable agreement with the tables; rather, although also scribal and computational errors appear to be present, the differences between the tables and my attempts at recomputation were clearly systematic, and not only caused by rounding of the latitude of the incidental horizon to an integer degree. Additional research will therefore be necessary to learn more about the exact method of calculation of the prorogations in the Horoscope of Iskandar Sultan.

Appendix F: Positions at birth and conception

Positions at the time of birth and initial prorogations. True positions of the Sun, the Moon, the lunar nodes and the planets at 4 hours after sunset (mean local time at Uzgand) on Sunday, 24 April 1384, and the initial prorogations for the date of Iskandar's birth from the tables on fols 23v–62v.

	longitude	latitude[97]	prorogations
Sun	1^s 12;38,45/40	+15;40,18	1^s 12;21
Moon	2^s 21;18,43	−4;44,52	2^s 16;58
Head	5^s 2;58,23/29		
Tail	11^s 2;58,23		
Saturn	2^s 15;28,35	−1; 8,48	2^s 14;26
Jupiter	2^s 2;27,59	−0;47,22	2^s 0;15
Mars	7^s 1;25,24	+0;39, 0	7^s 1;31
Venus	11^s 27;17,56	−0;22,25	11^s 28;21
Mercury	2^s 0;16,48	+1; 0,36	2^s 0;46
Lot of fortune	7^s 21;37,32		7^s 21;53
Ascendant	9^s 0;17,35		9^s 0;18
2nd house	10^s 6;12		10^s 19;52
3rd house	11^s 15;44		0^s 3;24
4th house	0^s 27; 6,24		0^s 27; 6
5th house	1^s 19;20		1^s 14; 3
6th house	2^s 10;14		2^s 1;59
7th house	3^s 0;17,35		3^s 0;28
8th house	4^s 6;12		4^s 19;52
9th house	5^s 15;44		6^s 3;24
Midheaven	6^s 27; 6,24		6^s 27; 6
11th house	7^s 19;20		7^s 14; 3
12th house	8^s 10;14		8^s 1;59

Positions at the time of conception. True positions of the planets and the lunar node at 1;46,23,52 hours after midnight (mean local time at Uzgand) on 13 July 1383 (according to the calculations given on fol. 4r: 22–26 for the Moon and on fol. 16r for all seven planets).

	longitude
Sun	3^s 27;27
Moon	9^s 0; 8,22
Head	5^s 18; 9,53
Saturn	2^s 13;11
Jupiter	1^s 21;49
Mars	2^s 11;15
Venus	4^s 18;37
Mercury	3^s 23; 8

[97] For the Sun: declination.

Abstract

In the year 813 Hijra (1411 CE) Imād al-munajjim Maḥmūd al-Kāshī, member of a well-known family of astronomers and mathematicians who worked for various rulers of the Timurid dynasty in modern-day Iran and Uzbekistan, composed a deluxe horoscope for one of those rulers, Iskandar Sultan (1384–1415). This work is extant in the manuscript London, Wellcome Library, Persian 474 and is particularly famous because of its magnificent colour depiction of Iskandar's birth horoscope. In the first part of the Horoscope, al-Kāshī provides extensive explanations (with occasional geometrical proofs) of the calculations that are needed in order to make the predictions on Iskandar's life found in the second part of the work, and the numerical results are summarised in large sets of tables. The main purpose of this article is to explain all calculations given in the text and to verify the numbers occurring in the manuscript. Since also explanations of basic concepts and ample references to the relevant literature are provided, it may at the same time serve as an introduction to Islamic mathematical astronomy and astrology.